

Achromatic colour vision with relative luminance

Mathematical hyperbel and potential functions

$$F_{ab}(x_r, a) = b \tanh(x_r/a) = b \frac{e^{x_r/a} - e^{-x_r/a}}{e^{x_r/a} + e^{-x_r/a}} \quad \begin{matrix} x_r = \log(L_r) \\ L_r = L/L_u \\ x_r \leq 0 \end{matrix} \quad [1]$$

$$\frac{dF_{ab}(x_r, a)}{dx_r} = \frac{4b}{a[e^{x_r/a} + e^{-x_r/a}]^2} \quad \begin{matrix} x_r = \ln L_r / \ln(10) \\ dx_r/dL_r = 1/(\ln(10)L_r) \\ m = 1/(\ln(10)a) \end{matrix} \quad [5]$$

$$\frac{L/dL}{(L/dL)_u} = \frac{4}{[e^{x_r/a} + e^{-x_r/a}]^2}; \quad \frac{dL}{dL_u} = \frac{[e^{x_r/a} + e^{-x_r/a}]^2 L}{4L_u} \quad [8]$$

$$\frac{L/dL}{(L/dL)_u} = \frac{4}{L_r^{2m} + 2 + L_r^{-2m}}; \quad \frac{dL}{dL_u} = \frac{(L_r^{2m} + 2 + L_r^{-2m})L}{4L_u} \quad [9]$$