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Achromatic colour vision with relative luminance
 Mathematical equations with hyperbel functions

$$F_{ab}(x_r, a) = b \tanh(x_r/a) = b \frac{e^{x_r/a} - e^{-x_r/a}}{e^{x_r/a} + e^{-x_r/a}} \quad x_r < 0 \quad [1]$$

$$\frac{dF_{ab}(x_r, a)}{dx_r} = \frac{4b}{a[e^{x_r/a} + e^{-x_r/a}]^2} \quad x_r = \ln L_r / \ln(10) \quad [2]$$

$$\frac{dF_{ab}(x_r, a)}{dL_r} = \frac{4bm}{[e^{x_r/a} + e^{-x_r/a}]^2 L_r} \quad dL_r / dL_u = 1 / (\ln(10) L_r) \quad [3]$$

$$\frac{L}{dL} = \frac{4bm}{[e^{x_r/a} + e^{-x_r/a}]^2} \quad dL = \frac{[e^{x_r/a} + e^{-x_r/a}]^2 L}{4bm} \quad [4]$$

Achromatic colour vision with relative luminance
Mathematical equations with hyperbel functions

$$F_{cb}(x_r, c) = b \tanh(x_r/c) = b \frac{e^{x_r/c} - e^{-x_r/c}}{e^{x_r/c} + e^{-x_r/c}} \quad \begin{matrix} x_r = \log(L_r) \\ L_r = L/L_u \\ x_r > 0 \end{matrix} \quad [1]$$

$$\frac{dF_{cb}(x_r, c)}{dx_r} = \frac{4b}{c[e^{x_r/c} + e^{-x_r/c}]^2} \quad \begin{matrix} x_r = \ln L / \ln(10) \\ dx_r/dL_r = 1/\ln(10) L_r \\ n = 1/(\ln(10)c) \end{matrix} \quad [5]$$

$$\frac{dF_{cb}(x_r, c)}{dx_r} \frac{dx_r}{dL_r} = \frac{4bm}{[e^{x_r/c} + e^{-x_r/c}]^2 L_r} \quad \begin{matrix} dF_{cb}(x_r, c) = 1 \\ dL/dL_r = dL/dL_r \end{matrix} \quad [6]$$

$$\frac{L}{dL} = \frac{4bm}{[e^{x_r/c} + e^{-x_r/c}]^2} \quad dL = \frac{[e^{x_r/c} + e^{-x_r/c}]^2 L}{4bm} \quad [7]$$

Achromatic colour vision with relative luminance
Mathematical hyperbel and potential functions

$$F_{ab}(x_r, a) = b \tanh(x_r/a) = b \frac{e^{x_r/a} - e^{-x_r/a}}{e^{x_r/a} + e^{-x_r/a}}$$

$$\frac{dF_{ab}(x_r, a)}{dx_r} = \frac{4b}{a[e^{x_r/a} + e^{-x_r/a}]^2}$$

$$\frac{L}{dL} = \frac{4bm}{[e^{x_r/a} + e^{-x_r/a}]^2}$$

$$\frac{dL}{dL_r} = \frac{[e^{x_r/a} + e^{-x_r/a}]^2 L}{4bm}$$

$$\frac{dL}{dL_r} = \frac{[L_r^m + L_r^{-m}]^2 L}{4bm}$$

Achromatic colour vision with relative luminance
 Mathematical hyperbel and potential functions

$$F_{cb}(x_r, c) = b \tanh(x_r/c) = b \frac{e^{x_r/c} - e^{-x_r/c}}{e^{x_r/c} + e^{-x_r/c}} \quad x_r = \log(L_r/L_u)$$

$$\frac{dF_{cb}(x_r, c)}{dx_r} = \frac{4b}{c[e^{x_r/c} + e^{-x_r/c}]^2} \quad x_r = \ln(L_r/L_u) \quad [1]$$

$$\frac{L}{dL} = \frac{4bn}{[e^{x_r/c} + e^{-x_r/c}]^2} \quad dL = \frac{[e^{x_r/c} + e^{-x_r/c}]^2 L}{4bn} \quad [7]$$

$$\frac{L}{dL} = \frac{4bn}{[L_r^n + L_r^{-n}]^2} \quad dL = \frac{[L_r^n + L_r^{-n}]^2 L}{4bn} \quad [8]$$

Achromatic colour vision with relative luminance
 Mathematical equations with hyperbel functions

$$F_{ab}(x_r, a) = b \tanh(x_r/a) = b \frac{e^{x_r/a} - e^{-x_r/a}}{e^{x_r/a} + e^{-x_r/a}} \quad x_r = \log(L_r) \quad L_r = L/L_u \quad x_r < 0 \quad [1]$$

$$\frac{dF_{ab}(x_r, a)}{dx_r} = \frac{4b}{a[e^{x_r/a} + e^{-x_r/a}]^2} \quad x_r = \ln(L_r/\ln(10)) \quad dx_r/dL_r = 1/(\ln(10)L_r) \quad m = 1/(\ln(10)a) \quad [5]$$

$$\frac{L}{dL} = \frac{4bm}{[e^{x_r/a} + e^{-x_r/a}]^2} \quad dL = \frac{a[e^{x_r/a} + e^{-x_r/a}]^2 L}{4bm} \quad [7]$$

$$\frac{L/dL}{(L/dL_u)} = \frac{4}{[e^{x_r/a} + e^{-x_r/a}]^2}; \quad \frac{dL}{dL_u} = \frac{[e^{x_r/a} + e^{-x_r/a}]^2 L}{4L_u} \quad [8]$$

$b = 0.1 - 3n \cdot \text{erf}(x_r/3n)$

Achromatic colour vision with relative luminance

Mathematical equations with hyperbel functions

$$F_{cb}(x_r, c) = b \tanh(x_r/c) = b \frac{e^{x_r/c} - e^{-x_r/c}}{e^{x_r/c} + e^{-x_r/c}} \quad \begin{matrix} x_r = \log(L_r/L_u) \\ L_r = L_u \end{matrix}$$

$$x_r > 0 \quad [1]$$

$$\frac{dF_{cb}(x_r, c)}{dx_r} = \frac{4b}{c[e^{x_r/c} + e^{-x_r/c}]^2} \quad \begin{matrix} x_r = \ln L_r / \ln(10) \\ dx_r/dL_r = 1 / (\ln(10) L_r) \\ n = 1 / (\ln(10) c) \end{matrix} \quad [5]$$

$$\frac{L}{dL} = \frac{4bn}{[e^{x_r/c} + e^{-x_r/c}]^2} \quad dL = \frac{c[e^{x_r/c} + e^{-x_r/c}]^2 L}{4bn} \quad [7]$$

$$\frac{L/dL}{(L/dL_u)} = \frac{4}{[e^{x_r/c} + e^{-x_r/c}]^2}; \quad \frac{dL}{dL_u} = \frac{[e^{x_r/c} + e^{-x_r/c}]^2 L}{4L_u} \quad [8]$$

Achromatic colour vision with relative luminance

Mathematical hyperbel and potential functions

$$F_{ab}(x_r, a) = b \tanh(x_r/a) = b \frac{e^{x_r/a} - e^{-x_r/a}}{e^{x_r/a} + e^{-x_r/a}} \quad \begin{array}{l} x_r = \log(L_r/L_u) \\ L_r = L_u e^{x_r} \\ x_r < 0 \end{array} \quad [1]$$

$$\frac{dF_{ab}(x_r, a)}{dx_r} = \frac{4b}{a[e^{x_r/a} + e^{-x_r/a}]^2} \quad \begin{array}{l} x_r = \ln(L_r/L_u) \\ dx_r/dL_r = 1/(\ln(10))L_r \\ m = 1/(ln(10))L_r \end{array} \quad [5]$$

$$\frac{dL}{dL_r} = \frac{4bm}{[e^{x_r/a} + e^{-x_r/a}]^2} \quad dL = \frac{[e^{x_r/a} + e^{-x_r/a}]^2 L}{4bm} \quad [7]$$

$$\frac{dL}{dL_r} = \frac{4bm}{[L_r^{2m} + 2 + L_r^{-2m}]^2} \quad dL = \frac{[L_r^{2m} + 2 + L_r^{-2m}] L}{4bm} \quad [8]$$

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Achromatic colour vision with relative luminance
Mathematical hyperbel and potential functions

$$F_{cb}(x_r, c) = b \tanh(x_r/c) = b \frac{e^{x_r/c} - e^{-x_r/c}}{e^{x_r/c} + e^{-x_r/c}}$$

$$\begin{aligned} x_r &= \log(L_r) \\ L_r &= L/L_u \\ x_r &\geq 0 \end{aligned}$$

$$[1]$$

$$\frac{dF_{cb}(x_r, c)}{dx_r} = \frac{4b}{c[e^{x_r/c} + e^{-x_r/c}]^2}$$

$$\begin{aligned} x_r &= \ln(L_r/\ln(10)) \\ dx_r/dL_r &= 1/(\ln(10)L_r) \\ n &= 1/(10(c)) \end{aligned}$$

$$[5]$$

$$\frac{L}{dL} = \frac{4bn}{[e^{x_r/c} + e^{-x_r/c}]^2} \quad dL = \frac{[e^{x_r/c} + e^{-x_r/c}]^2 L}{4bn}$$

$$[7]$$

$$\frac{L}{dL} = \frac{4bn}{[L_r^{2n} + 2 + L_r^{-2n}]^2} \quad dL = \frac{[L_r^{2n} + 2 + L_r^{-2n}] L}{4bn}$$

$$[8]$$

Achromatic colour vision with relative luminance
 Mathematical equations with hyperbel functions

$$F_{ab}(x_r, a) = b \tanh(x_r/a) = b \frac{e^{x_r/a} - e^{-x_r/a}}{e^{x_r/a} + e^{-x_r/a}} \quad x_r < \log(L_r)$$

$$\frac{dF_{ab}(x_r, a)}{dx_r} = \frac{4b}{a[e^{x_r/a} + e^{-x_r/a}]^2} \quad \begin{aligned} x_r &= \ln L_r / \ln(10) \\ dx_r/dL_r &= 1/(\ln(10)L_r) \end{aligned}$$

$$\frac{L/dL}{(L/dL)_u} = \frac{4}{[e^{x_r/a} + e^{-x_r/a}]^2}; \quad \frac{dL}{dL_u} = \frac{[e^{x_r/a} + e^{-x_r/a}]^2 L}{4 L_u} \quad [8]$$

$$\frac{L/dL}{(L/dL)_u} = 1 \text{ for } \begin{cases} L=L_u \\ x_r=0 \end{cases} \quad \frac{dL}{dL_u} = 1 \text{ for } \begin{cases} L=L_u \\ x_r=0 \end{cases} \quad [9]$$

Achromatic colour vision with relative luminance
 Mathematical equations with hyperbel functions

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$$\begin{aligned} x_r &= \log(L_u/L_r) \\ L_r &= L_u e^{x_r/c} \\ x_r &\geq 0 \quad [1] \end{aligned}$$

$$\frac{dF_{cb}(x_r, c)}{dx_r} = \frac{4b}{c[e^{x_r/c} + e^{-x_r/c}]^2} \quad \begin{aligned} x_r &= \ln L_u / \ln(10) \\ \frac{dx_r}{dL_r} &= 1 / (\ln(10) L_r) \\ n &= 1 / (\ln(10) c) \quad [5] \end{aligned}$$

$$\frac{L/dL}{(L/dL)_u} = \frac{4}{[e^{x_r/c} + e^{-x_r/c}]^2}, \quad \frac{dL}{dL_u} = \frac{[e^{x_r/c} + e^{-x_r/c}]^2 L}{4 L_u} \quad [8]$$

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Achromatic colour vision with relative luminance
 Mathematical hyperbel and potential functions

$$F_{ab}(x_r, a) = b \tanh(x_r/a) = b \frac{e^{x_r/a} - e^{-x_r/a}}{e^{x_r/a} + e^{-x_r/a}} \quad x_r = \log(L_r/L_u)$$

$$\frac{dF_{ab}(x_r, a)}{dx_r} = \frac{4b}{a[e^{x_r/a} + e^{-x_r/a}]^2} \quad x_r = \ln(L_r/L_u) \quad \frac{dx_r}{dL_r} = 1/(ln(10)L_r) \quad m = 1/(ln(10)a) \quad [5]$$

$$\frac{L/dL}{(L/dL)_u} = \frac{4}{[e^{x_r/a} + e^{-x_r/a}]^2}; \quad \frac{dL}{dL_u} = \frac{[e^{x_r/a} + e^{-x_r/a}]^2 L}{4L_u} \quad [8]$$

$$\frac{L/dL}{(L/dL)_u} = \frac{4}{[L_r^m + L_r^{-m}]^2}; \quad \frac{dL}{dL_u} = \frac{[L_r^m + L_r^{-m}]^2 L}{4L_u} \quad [9]$$

Achromatic colour vision with relative luminance
Mathematical hyperbel and potential functions

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$$\frac{dF_{cb}(x_r, c)}{dx_r} = \frac{4b}{c[e^{x_r/c} + e^{-x_r/c}]^2} \quad x_r = \ln(L_r/L_u) \quad dx_r/dL_r = 1/\ln(10)L_r$$

$$\frac{L/dL}{(L/dL)_u} = \frac{4}{[e^{x_r/c} + e^{-x_r/c}]^2}; \quad \frac{dL}{dL_u} = \frac{[e^{x_r/c} + e^{-x_r/c}]^2 L}{4L_u} \quad [8]$$

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Achromatic colour vision with relative luminance
Mathematical equations with hyperbel functions

$$F_{ab}(x_r, a) = b \tanh(x_r/a) = b \frac{e^{x_r/a} - e^{-x_r/a}}{e^{x_r/a} + e^{-x_r/a}}, \quad x_r < 0 \quad [1]$$

$$\frac{dF_{ab}(x_r, a)}{dx_r} = \frac{4b}{a[e^{x_r/a} + e^{-x_r/a}]^2}, \quad x_r = \ln L_r / \ln(10), \quad dx_r/dL_r = 1/(\ln(10)L_r), \quad m = 1/(\ln(10)a) \quad [5]$$

$$\frac{L/dL}{(L/dL)_u} = \frac{4}{[e^{x_r/a} + e^{-x_r/a}]^2}; \quad \frac{dL}{dL_u} = \frac{[e^{x_r/a} + e^{-x_r/a}]^2 L}{4L_u} \quad [8]$$

$$\frac{L/dL}{(L/dL)_u} = 1 \text{ for } \begin{cases} L_r = 1 \\ x_r = 0 \end{cases} \quad \frac{dL}{dL_u} = 1 \text{ for } \begin{cases} L_r = 1 \\ x_r = 0 \end{cases} \quad [9]$$

Achromatic colour vision with relative luminance
Mathematical equations with hyperbel functions

$$F_{cb}(x_r, c) = b \tanh(x_r/c) = b \frac{e^{x_r/c} - e^{-x_r/c}}{e^{x_r/c} + e^{-x_r/c}} \quad x_r = \log(L_r) \quad L_r = L/L_u \quad x_r = 0 \quad [1]$$

$$\frac{dF_{cb}(x_r, c)}{dx_r} = \frac{4b}{c[e^{x_r/c} + e^{-x_r/c}]^2} \quad x_r = \ln L_r / \ln(10) \quad \frac{dx_r/dL_r}{dL_r} = 1/(\ln(10)L_r) \quad n = 1/(ln(10)c) \quad [5]$$

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$$\frac{L/dL}{(L/L_u)_u} = 1 \text{ for } \begin{cases} L_r = 1 \\ x_r = 0 \end{cases} \quad \frac{dL}{dL_u} = 1 \text{ for } \begin{cases} L_r = 1 \\ x_r = 0 \end{cases} \quad [9]$$

Achromatic colour vision with relative luminance
 Mathematical hyperbel and potential functions

$$F_{ab}(x_r, a) = b \tanh(x_r/a) = b \frac{e^{x_r/a} - e^{-x_r/a}}{e^{x_r/a} + e^{-x_r/a}} \quad x_r = \log(L_r/L_u) \quad L_r = L/L_u$$

$$\frac{dF_{ab}(x_r, a)}{dx_r} = \frac{4b}{a[e^{x_r/a} + e^{-x_r/a}]^2} \quad x_r = \ln(L_r/\ln(10)) \quad dx_r/dL_r = 1/(ln(10)L_r)$$

$$\frac{L/dL}{(L/dL)_u} = \frac{4}{[e^{x_r/a} + e^{-x_r/a}]^2}; \quad \frac{dL}{dL_u} = \frac{[e^{x_r/a} + e^{-x_r/a}]^2 L}{4L_u} \quad [8]$$

$$\frac{L/dL}{(L/dL)_u} = \frac{4}{L_r^{2m} + 2 + L_r^{-2m}}; \quad \frac{dL}{dL_u} = \frac{(L_r^{2m} + 2 + L_r^{-2m})L}{4L_u} \quad [9]$$

Achromatic colour vision with relative luminance	
Mathematical hyperbel and potential functions	
$F_{Cb}(x_r, c) = b \tanh(x_r/c) - b \frac{e^{x_r/c} - e^{-x_r/c}}{e^{x_r/c} + e^{-x_r/c}}$	$x_r = \log(L_r)$ $L_r = L/L_u$ $x_r > 0$ [1]
$\frac{dF_{Cb}(x_r, c)}{dx_r} = \frac{4b}{c[e^{x_r/c} + e^{-x_r/c}]^2}$	$x_r - \ln x_r / \ln(10)$ $dx_r/dL_r = 1/(\ln(10)L_r)$ $n = 1/(ln(10)c)$ [5]
$\frac{L/dL}{(L/dL)_u} = \frac{4}{[e^{x_r/c} + e^{-x_r/c}]^2}; \frac{dL}{dL_u} = \frac{[e^{x_r/c} + e^{-x_r/c}]^2 L}{4L_u}$	[8]
$\frac{L/dL}{(L/dL)_u} = \frac{4}{L_r^{2n} + 2 + L_r^{-2n}}; \frac{dL}{dL_u} = \frac{(L_r^{2n} + 2 + L_r^{-2n})L}{4L_u}$	[9]

TUB-test chart hey0; Model of normalized receptor-response functions $F_{ab}(x_r)$ and $F_{cb}(x_r)$

Calculation of derivations $F'_{ab}(x_r)$, $F'_{cb}(x_r)$, of contrasts $L/\Delta L$, and discriminations $(\Delta L)_{ab}$, $(\Delta L)_{cb}$