

# Achromatic colour vision with relative luminance

## Mathematical hyperbel and potential functions

$$F_{cb}(x_r, c) = b \tanh(x_r/c) = b \frac{e^{x_r/c} - e^{-x_r/c}}{e^{x_r/c} + e^{-x_r/c}} \quad \begin{matrix} x_r = \log(L_r) \\ L_r = L/L_u \\ x_r \geq 0 \end{matrix} \quad [1]$$

$$\frac{dF_{cb}(x_r, c)}{dx_r} = \frac{4b}{c[e^{x_r/c} + e^{-x_r/c}]^2} \quad \begin{matrix} x_r = \ln L_r / \ln(10) \\ dx_r/dL_r = 1/(\ln(10)L_r) \\ n = 1/(\ln(10)c) \end{matrix} \quad [5]$$

$$\frac{L}{dL} = \frac{4bn}{[e^{x_r/c} + e^{-x_r/c}]^2} \quad dL = \frac{[e^{x_r/c} + e^{-x_r/c}]^2 L}{4bn} \quad [7]$$

$$\frac{L}{dL} = \frac{4bn}{[L_r^{2n} + 2 + L_r^{-2n}]} \quad dL = \frac{[L_r^{2n} + 2 + L_r^{-2n}] L}{4bn} \quad [8]$$