Mathematical equations of hyperbel functions

See: *Papula, L., (2003), Mathematische Formelsammlung, Vieweg*

$$F(x)=\tanh(x)=\frac{e^{x}-e^{-x}}{e^{x}+e^{-x}}=\frac{u(x)}{v(x)} \qquad u'(x)=v(x),\ v'(x)=u(x) \qquad [1]$$

$$F'(x)=\frac{u'(x)\,v(x)-u(x)\,v'(x)}{v^2(x)}=\frac{v^2(x)-u^2(x)}{v^2(x)} \qquad [2]$$

$$F'(x)=\frac{[e^{x}+e^{-x}][e^{x}+e^{-x}]-[e^{x}-e^{-x}][e^{x}-e^{-x}]}{[e^{x}+e^{-x}]^2} \qquad [3]$$

$$F'(x)=\frac{4}{[e^{x}+e^{-x}]^2}=\frac{1}{\cosh^2(x)} \qquad [4]$$

hex41−2n, eeo31−2n

Mathematical equations of hyperbel functions

See: *Papula, L., (2003), Mathematische Formelsammlung, Vieweg*

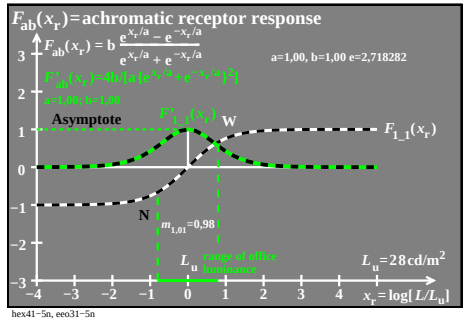
$$F(x/a)=\tanh(x/a)=\frac{e^{x/a}-e^{-x/a}}{e^{x/a}+e^{-x/a}}=\frac{u(x/a)}{v(x/a)} \qquad [1]$$

$$F'(x/a)=\frac{u'(x/a)v(x/a)-u(x/a)v'(x/a)}{v^2(x/a)} \qquad [2]$$

$$F'(x/a)=\frac{v^2(x/a)-u^2(x/a)}{a\,v^2(x/a)} \qquad [3]$$

$$F'(x/a)=\frac{4}{a\,[e^{x/a}+e^{-x/a}]^2}=\frac{1}{a\cosh^2(x/a)} \qquad [4]$$

hex41−4n, eeo31−4n



hex41−5n, eeo31−5n

Mathematical equations of hyperbel functions

See: *Papula, L., (2003), Mathematische Formelsammlung, Vieweg*

$$F_{ab}(x/a)=b\tanh(x/a)=b\frac{e^{x/a}-e^{-x/a}}{e^{x/a}+e^{-x/a}}=b\frac{u(x/a)}{v(x/a)} \qquad [1]$$

$$F'_{ab}(x/a)=b\frac{u'(x/a)\,v(x/a)-u(x/a)\,v'(x/a)}{v^2(x/a)} \qquad [2]$$

$$F'_{ab}(x/a)=b\frac{v^2(x/a)-u^2(x/a)}{a\,v^2(x/a)} \qquad [3]$$

$$F'_{ab}(x/a)=\frac{4b}{a\,[e^{x/a}+e^{-x/a}]^2}=\frac{b}{a\cosh^2(x/a)} \qquad [4]$$

hex41−2n eeo41−2n

Mathematical equations of hyperbel functions

See: *Papula, L., (2003), Mathematische Formelsammlung, Vieweg*

$$F_{1b}(x)=b\tanh(x)=b\frac{e^{x}-e^{-x}}{e^{x}+e^{-x}}=b\frac{u(x)}{v(x)} \qquad [1]$$

$$F'_{1b}(x)=b\frac{u'(x)\,v(x)-u(x)\,v'(x)}{v^2(x)} \qquad [2]$$

$$F'_{1b}(x)=b\frac{v^2(x)-u^2(x)}{a\,v^2(x)} \qquad [3]$$

$$F'_{1b}(x)=\frac{4b}{[e^{x}+e^{-x}]^2}=\frac{b}{\cosh^2(x)} \qquad [4]$$

hex41−6n, eeo31−6n

Mathematical equations of hyperbel functions

See: *Papula, L., (2003), Mathematische Formelsammlung, Vieweg*

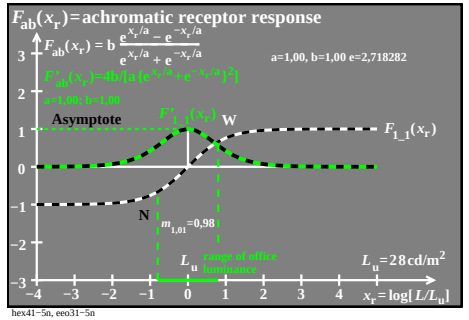
$$F_{ab}(x/a)=b\tanh(x/a)=b\frac{e^{x/a}-e^{-x/a}}{e^{x/a}+e^{-x/a}}=b\frac{u(x/a)}{v(x/a)} \qquad [1]$$

$$F'_{ab}(x/a)=b\frac{u'(x/a)\,v(x/a)-u(x/a)\,v'(x/a)}{v^2(x/a)} \qquad [2]$$

$$F'_{ab}(x/a)=b\frac{v^2(x/a)-u^2(x/a)}{a\,v^2(x/a)} \qquad [3]$$

$$F'_{ab}(x/a)=\frac{4b}{a\,[e^{x/a}+e^{-x/a}]^2}=\frac{b}{a\cosh^2(x/a)} \qquad [4]$$

hex41−8n, eeo31−8n



hex41−5n, eeo31−5n

Mathematical equations of hyperbel functions

See: *Papula, L., (2003), Mathematische Formelsammlung, Vieweg*

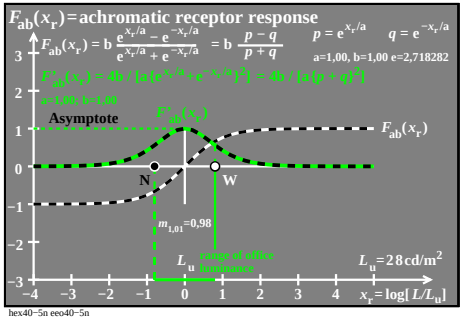
$$F_{ab}(x/a)=b\tanh(x/a)=b\frac{e^{x/a}-e^{-x/a}}{e^{x/a}+e^{-x/a}}=b\frac{u(x/a)}{v(x/a)} \qquad [1]$$

$$F'_{ab}(x/a)=b\frac{u'(x/a)\,v(x/a)-u(x/a)\,v'(x/a)}{v^2(x/a)} \qquad [2]$$

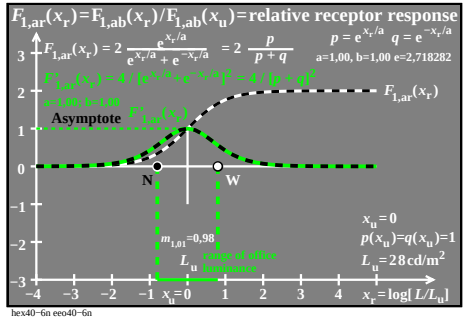
$$F'_{ab}(x/a)=b\frac{v^2(x/a)-u^2(x/a)}{a\,v^2(x/a)} \qquad [3]$$

$$F'_{ab}(x/a)=\frac{4b}{a\,[e^{x/a}+e^{-x/a}]^2}=\frac{b}{a\cosh^2(x/a)} \qquad [4]$$

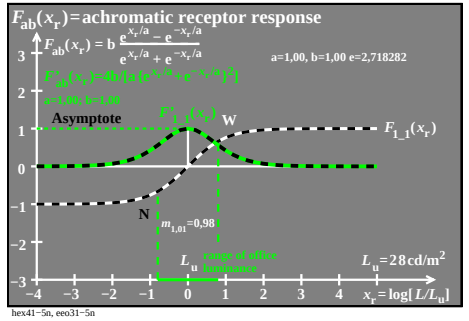
hex41−4n eeo41−4n



hex40−5n eeo40−5n



hex40−6n eeo40−6n



hex41−5n, eeo31−5n

Mathematical equations of hyperbel functions

See: *Papula, L., (2003), Mathematische Formelsammlung, Vieweg*

$$F_{ab}(x/a)=b\tanh(x/a)=b\frac{e^{x/a}-e^{-x/a}}{e^{x/a}+e^{-x/a}} \qquad [1]$$

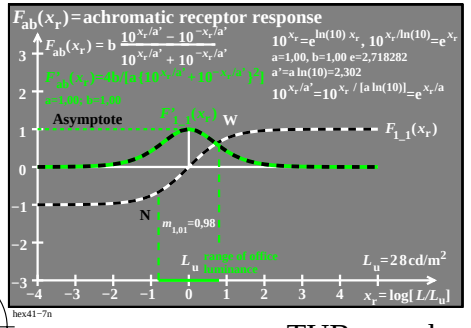
$$\frac{dF_{ab}(x/a)}{dx}=\frac{4b}{a[e^{x/a}+e^{-x/a}]^2}=\frac{b}{a\cosh^2(x/a)} \qquad [4]$$

$$\frac{dF_{ab}(x_r/a)}{dx_r}=\frac{4b}{a[e^{x_r/a}+e^{-x_r/a}]^2} \qquad x_r=\log(L/L_u) \quad [5]$$

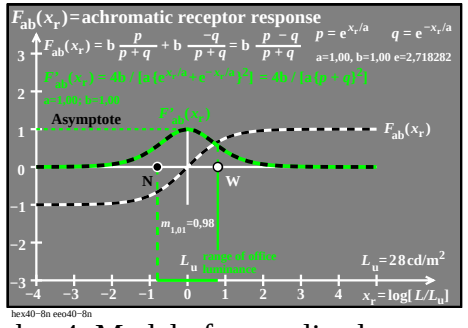
$$dx_r/dL=\ln(10)/L$$

$$\frac{dF_{ab}(x_r/a)}{dx_r}\,\frac{dx_r}{dL}=\frac{4b}{a[e^{x_r/a}+e^{-x_r/a}]^2}\,\frac{\ln(10)}{L} \qquad [6]$$

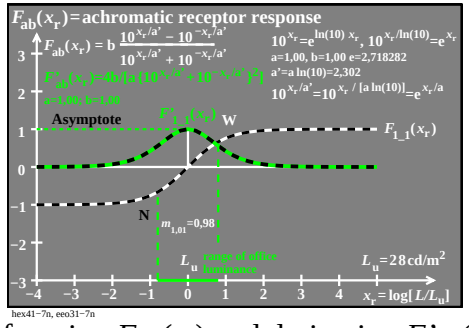
hex41−6n eeo41−6n



hex41−7n



hex40−8n eeo40−8n



hex41−7n, eeo31−7n

Mathematical equations of hyperbel functions

See: *Papula, L., (2003), Mathematische Formelsammlung, Vieweg*

$$F_{ab}(x_r/a)=b\tanh(x_r/a)=b\frac{e^{x_r/a}-e^{-x_r/a}}{e^{x_r/a}+e^{-x_r/a}} \qquad [1]$$

$$\frac{dF_{ab}(x_r/a)}{dx_r}=\frac{4b}{a[e^{x_r/a}+e^{-x_r/a}]^2} \qquad x_r=\log(L/L_u) \quad [5]$$

$$dx_r/dL=\ln(10)/L$$

$$\frac{dF_{ab}(x_r/a)}{dx_r}\,\frac{dx_r}{dL}=\frac{4b}{a[e^{x_r/a}+e^{-x_r/a}]^2}\,\frac{\ln(10)}{L} \qquad [6]$$

$$\frac{L}{dL}=\frac{4b\ln(10)}{a[e^{x_r/a}+e^{-x_r/a}]^2} \qquad dL=\frac{a[e^{x_r/a}+e^{-x_r/a}]^2 L}{4b\ln(10)} \qquad [7]$$

hex41−8n eeo41−8n

TUB-test chart hex4; Model of normalized response function $F_{ab}(x_r)$ and derivation $F'_{ab}(x_r)$
Mathematical calculation of the derivation $F'_{ab}(x_r)$, of the contrast $L/\Delta L$, and the discrimination ΔL