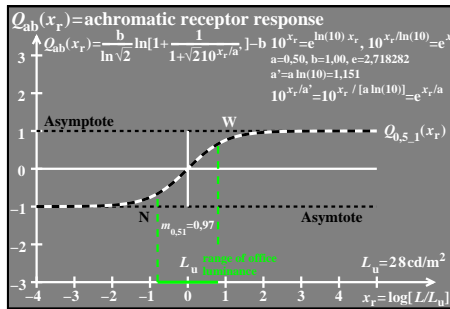
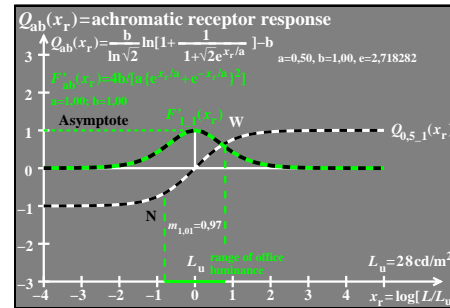


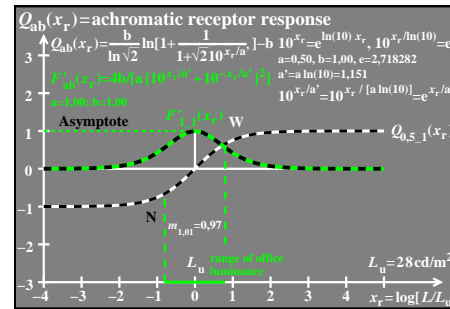
Achromatic receptor-response function
 $Q_{ab}[x_r/a]$ for $a=0.5$ and $b=1.0$
 with $x_r = \log [L/L_u]$ (L =test luminance)
 L_u =surround luminance
 $Q_{ab}[x_r/a] = \frac{b}{\ln \sqrt{2}} \ln \left[\frac{1}{1 + \sqrt{2} e^{(x_r/a)}} \right] - b$
function values for $b=1$ and any $a>0$:
 $Q_{a1}[x_r/a \rightarrow -\infty] = -1 \quad x = \log L, u = \log L_u$
 $Q_{a1}[x_r/a = 0] = 0 \quad x_r = \log [L/L_u]$
 $Q_{a1}[x_r/a \rightarrow +\infty] = +1 \quad = x - u$



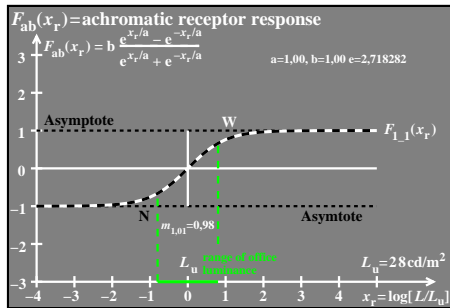
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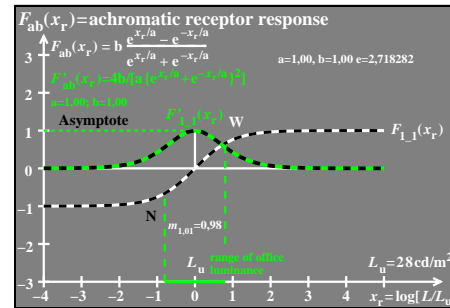
Mathematical equations of hyperbel functions
 See: Papula, L., (2003), *Mathematische Formelsammlung*, Vieweg
 $F(x) = \tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{u(x)}{v(x)} \quad u'(x) = v(x) \quad v'(x) = u(x)$ [1]
 $F'(x) = \frac{u'(x)v(x) - u(x)v'(x)}{v^2(x)} = \frac{v^2(x) - u^2(x)}{v^2(x)}$ [2]
 $F'(x) = \frac{[e^x + e^{-x}][e^x + e^{-x}] - [e^x - e^{-x}][e^x - e^{-x}]}{[e^x + e^{-x}]^2}$ [3]
 $F'(x) = \frac{4}{[e^x + e^{-x}]^2} = \frac{1}{\cosh^2(x)}$ [4]



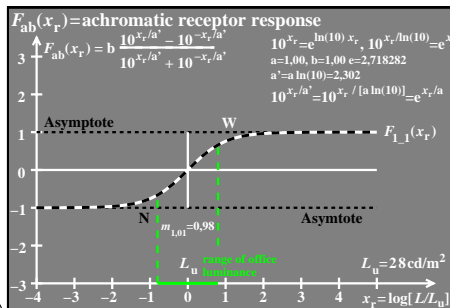
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 $F(x/a) = \tanh(x/a) = \frac{e^{x/a} - e^{-x/a}}{e^{x/a} + e^{-x/a}} = \frac{u(x/a)}{v(x/a)}$ [1]
 $F'(x/a) = \frac{u'(x/a)v(x/a) - u(x/a)v'(x/a)}{v^2(x/a)}$ [2]
 $F'(x/a) = \frac{v^2(x/a) - u^2(x/a)}{a v^2(x/a)}$ [3]
 $F'(x/a) = \frac{4}{a [e^{x/a} + e^{-x/a}]^2} = \frac{1}{a \cosh^2(x/a)}$ [4]



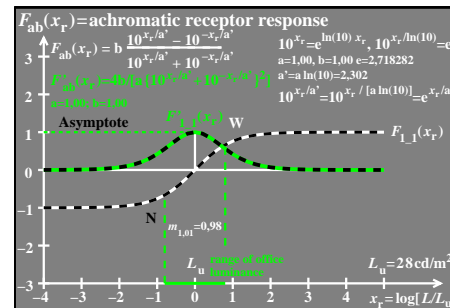
Mathematical equations of hyperbel functions
 See: Papula, L., (2003), *Mathematische Formelsammlung*, Vieweg
 $\sinh(x) = \frac{e^x - e^{-x}}{2}$ [1], $\cosh(x) = \frac{e^x + e^{-x}}{2}$ [2]
 $\tanh(x) = \frac{\sinh(x)}{\cosh(x)} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$ [3]
 $\tanh(x/2) = \frac{\sinh(x)}{\cosh(x) + 1} = \frac{\cosh(x) + 1}{\sinh(x)} = \frac{e^{x/2} - e^{-x/2}}{e^{x/2} + e^{-x/2}}$ [4]
 $\sinh^2(x) + \cosh^2(x) = 1$ [5]



Mathematical equations of hyperbel functions
 See: Papula, L., (2003), *Mathematische Formelsammlung*, Vieweg
 $F_{1b}(x) = b \tanh(x) = b \frac{e^x - e^{-x}}{e^x + e^{-x}} = b \frac{u(x)}{v(x)}$ [1]
 $F'_{1b}(x) = b \frac{u'(x)v(x) - u(x)v'(x)}{v^2(x)}$ [2]
 $F'_{1b}(x) = b \frac{v^2(x) - u^2(x)}{a v^2(x)}$ [3]
 $F'_{1b}(x) = \frac{4b}{[e^x + e^{-x}]^2} = \frac{b}{\cosh^2(x)}$ [4]



Mathematical equations of hyperbel functions
 See: Papula, L., (2003), *Mathematische Formelsammlung*, Vieweg
 $\sinh(x) = \frac{10^{x_r/a'} - 10^{-x_r/a'}}{2}$ [1], $\cosh(x) = \frac{10^{x_r/a'} + 10^{-x_r/a'}}{2}$ [2]
 $\tanh(x) = \frac{\sinh(x)}{\cosh(x)} = \frac{10^{x_r/a'} - 10^{-x_r/a'}}{10^{x_r/a'} + 10^{-x_r/a'}}$ [3]
 $\tanh(x/2) = \frac{\sinh(x)}{\cosh(x) + 1} = \frac{\cosh(x) + 1}{\sinh(x)} = \frac{10^{x_r/2a'} - 10^{-x_r/2a'}}{10^{x_r/2a'} + 10^{-x_r/2a'}}$ [4]
 $\sinh^2(x) + \cosh^2(x) = 1$ [5]



Mathematical equations of hyperbel functions
 See: Papula, L., (2003), *Mathematische Formelsammlung*, Vieweg
 $F_{ab}(x/a) = b \tanh(x/a) = b \frac{e^{x/a} - e^{-x/a}}{e^{x/a} + e^{-x/a}} = b \frac{u(x/a)}{v(x/a)}$ [1]
 $F'_{ab}(x/a) = b \frac{u'(x/a)v(x/a) - u(x/a)v'(x/a)}{v^2(x/a)}$ [2]
 $F'_{ab}(x/a) = b \frac{v^2(x/a) - u^2(x/a)}{a v^2(x/a)}$ [3]
 $F'_{ab}(x/a) = \frac{4b}{a [e^{x/a} + e^{-x/a}]^2} = \frac{b}{a \cosh^2(x/a)}$ [4]