

Mathematical equations of hyperbel functions

See: Papula, L., (2003), *Mathematische Formelsammlung*, Vieweg

$$F(x) = \tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{u(x)}{v(x)} \quad \begin{array}{l} u'(x) = v(x) \\ v'(x) = u(x) \end{array} \quad [1]$$

$$F'(x) = \frac{u'(x) v(x) - u(x) v'(x)}{v^2(x)} = \frac{v^2(x) - u^2(x)}{v^2(x)} \quad [2]$$

$$F'(x) = \frac{[e^x + e^{-x}][e^x + e^{-x}] - [e^x - e^{-x}][e^x - e^{-x}]}{[e^x + e^{-x}]^2} \quad [3]$$

$$F'(x) = \frac{4}{[e^x + e^{-x}]^2} = \frac{1}{\cosh^2(x)} \quad [4]$$