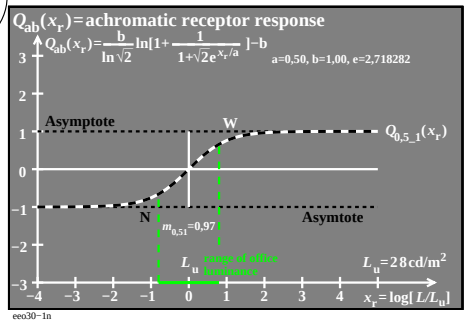
$Q_{ab}(x_r)$=achromatic receptor response

$Q_{ab}(x_r)=\frac{b}{\ln\sqrt{2}}\ln[1+\frac{1}{1+\sqrt{2}e^{x_r/a}}]-b$ a=0,50, b=1,00, e=2,718282




eeo30−1n

**Achromatic receptor-response function $Q_{ab}[x_r/a]$ for a=0,5 and b=1,0**

with $x_r$=log [$L/L_u$] ($L$=test luminance)
$L_u$=surround luminance

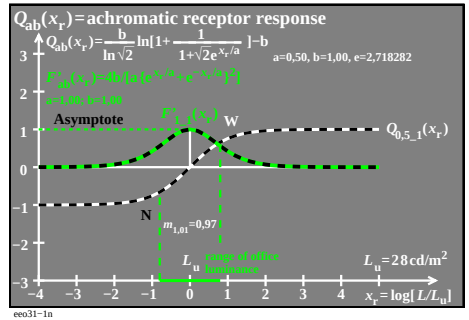
$$Q_{ab}[x_r/a]=\frac{b}{\ln\sqrt{2}}\ln[\frac{1}{1+\sqrt{2}e^{(x_r/a)}}]-b$$

function values for $b$=1 and any $a$>0 :

$Q_{a1}[x_r/a\to-\infty]=-1$ $\quad x=\log L,\ u=\log L_u$
$Q_{a1}[x_r/a=0]=0$ $\quad x_r=\log[L/L_u]$
$Q_{a1}[x_r/a\to+\infty]=+1$ $\quad =x-u$

eeo30−2n

$Q_{ab}(x_r)$=achromatic receptor response




eee31−1n

**Mathematical equations of hyperbel functions**

See: *Papula, L., (2003), Mathematische Formelsammlung, Vieweg*

$$F(x)=\tanh(x)=\frac{e^x-e^{-x}}{e^x+e^{-x}}=\frac{u(x)}{v(x)}\quad u'(x)=v(x)\ \ v'(x)=u(x)\quad[1]$$

$$F'(x)=\frac{u'(x)\,v(x)-u(x)\,v'(x)}{v^2(x)}=\frac{v^2(x)-u^2(x)}{v^2(x)}\quad[2]$$

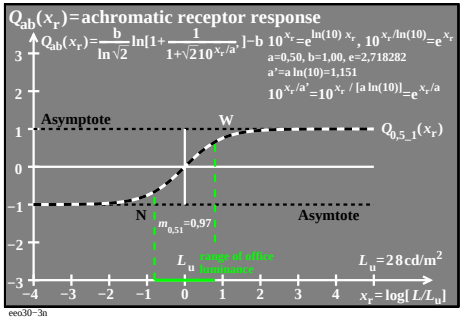
$$F'(x)=\frac{[e^x+e^{-x}][e^x+e^{-x}]-[e^x-e^{-x}][e^x-e^{-x}]}{[e^x+e^{-x}]^2}\quad[3]$$

$$F'(x)=\frac{4}{[e^x+e^{-x}]^2}=\frac{1}{\cosh^2(x)}\quad[4]$$

eee31−2n

$Q_{ab}(x_r)$=achromatic receptor response

$Q_{ab}(x_r)=\frac{b}{\ln\sqrt{2}}\ln[1+\frac{1}{1+\sqrt{2}10^{x_r/a'}}]-b$ $10^{x_r}=e^{\ln(10)\,x_r}$, $10^{x_r/\ln(10)}=e^{x_r}$
a=0,50, b=1,00, e=2,718282
a'=a ln(10)=1,151
$10^{x_r/a'}=10^{x_r/[a\ln(10)]}=e^{x_r/a}$




eee30−3n

**Achromatic receptor-response function $Q_{ab}[x_r/a]$ for a=0,5 and b=1,0**

with $x_r$=log [$L/L_u$] ($L$=test luminance)
$L_u$=surround luminance

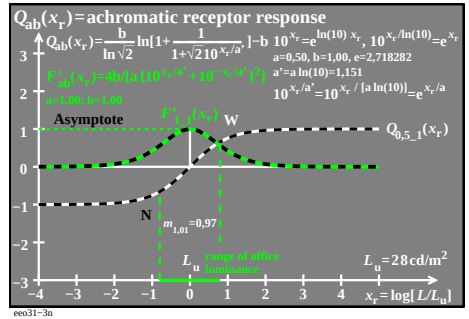
$$Q_{ab}[x_r/a]=\frac{b}{\ln\sqrt{2}}\ln[\frac{1}{1+\sqrt{2}e^{(x_r/a)}}]-b$$

function values for $b$=1 and any $a$>0 :

$Q_{a1}[x_r/a\to-\infty]=-1$ $\quad x=\log L,\ u=\log L_u$
$Q_{a1}[x_r/a=0]=0$ $\quad x_r=\log[L/L_u]$
$Q_{a1}[x_r/a\to+\infty]=+1$ $\quad =x-u$

eee30−4n

$Q_{ab}(x_r)$=achromatic receptor response




eee31−3n

**Mathematical equations of hyperbel functions**

See: *Papula, L., (2003), Mathematische Formelsammlung, Vieweg*

$$F(x/a)=\tanh(x/a)=\frac{e^{x/a}-e^{-x/a}}{e^{x/a}+e^{-x/a}}=\frac{u(x/a)}{v(x/a)}\quad[1]$$

$$F'(x/a)=\frac{u'(x/a)v(x/a)-u(x/a)v'(x/a)}{v^2(x/a)}\quad[2]$$

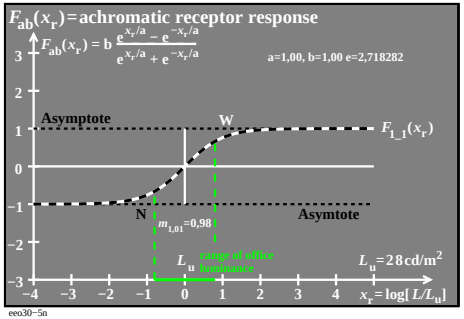
$$F'(x/a)=\frac{v^2(x/a)-u^2(x/a)}{a\,v^2(x/a)}\quad[3]$$

$$F'(x/a)=\frac{4}{a\,[e^{x/a}+e^{-x/a}]^2}=\frac{1}{a\cosh^2(x/a)}\quad[4]$$

eee31−4n

$F_{ab}(x_r)$=achromatic receptor response

$F_{ab}(x_r)=b\,\frac{e^{x_r/a}-e^{-x_r/a}}{e^{x_r/a}+e^{-x_r/a}}$ a=1,00, b=1,00 e=2,718282




eee30−5n

**Mathematical equations of hyperbel functions**

See: *Papula, L., (2003), Mathematische Formelsammlung, Vieweg*

$$\sinh(x)=\frac{e^x-e^{-x}}{2}\ [1],\quad \cosh(x)=\frac{e^x+e^{-x}}{2}\quad[2]$$

$$\tanh(x)=\frac{\sinh(x)}{\cosh(x)}=\frac{e^x-e^{-x}}{e^x+e^{-x}}\quad[3]$$

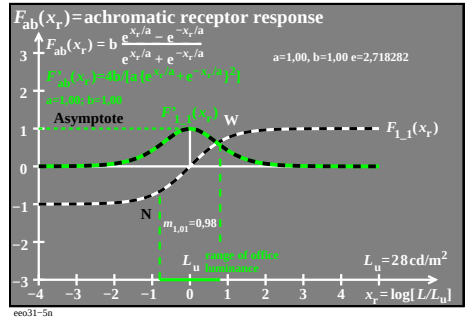
$$\tanh(x/2)=\frac{\sinh(x)}{\cosh(x)+1}=\frac{\cosh(x)+1}{\sinh(x)}=\frac{e^{x/2}-e^{-x/2}}{e^{x/2}+e^{-x/2}}\quad[4]$$

$$\sinh^2(x)+\cosh^2(x)=1\quad[5]$$

eee30−6n

$F_{ab}(x_r)$=achromatic receptor response

$F_{ab}(x_r)=b\,\frac{e^{x_r/a}-e^{-x_r/a}}{e^{x_r/a}+e^{-x_r/a}}$ a=1,00, b=1,00 e=2,718282




eee31−5n

**Mathematical equations of hyperbel functions**

See: *Papula, L., (2003), Mathematische Formelsammlung, Vieweg*

$$F_{1b}(x)=b\tanh(x)=b\frac{e^x-e^{-x}}{e^x+e^{-x}}=b\frac{u(x)}{v(x)}\quad[1]$$

$$F'_{1b}(x)=b\frac{u'(x)\,v(x)-u(x)\,v'(x)}{v^2(x)}\quad[2]$$

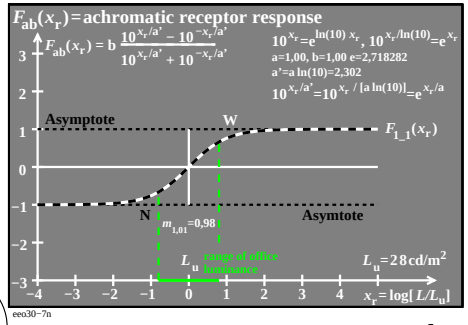
$$F'_{1b}(x)=b\frac{v^2(x)-u^2(x)}{a\,v^2(x)}\quad[3]$$

$$F'_{1b}(x)=\frac{4b}{[e^x+e^{-x}]^2}=\frac{b}{\cosh^2(x)}\quad[4]$$

eee31−6n

$F_{ab}(x_r)$=achromatic receptor response

$F_{ab}(x_r)=b\,\frac{10^{x_r/a'}-10^{-x_r/a'}}{10^{x_r/a'}+10^{-x_r/a'}}$ $10^{x_r}=e^{\ln(10)\,x_r}$, $10^{x_r/\ln(10)}=e^{x_r}$
a=1,00, b=1,00 e=2,718282
a'=a ln(10)=2,302
$10^{x_r/a'}=10^{x_r/[a\ln(10)]}=e^{x_r/a}$




eee30−7n

**Mathematical equations of hyperbel functions**

See: *Papula, L., (2003), Mathematische Formelsammlung, Vieweg*

$$\sinh(x)=\frac{10^{x_r/a'}-10^{-x_r/a'}}{2}\ [1],\quad \cosh(x)=\frac{10^{x_r/a'}+10^{-x_r/a'}}{2}\quad[2]$$

$$\tanh(x)=\frac{\sinh(x)}{\cosh(x)}=\frac{10^{x_r/a'}-10^{-x_r/a'}}{10^{x_r/a'}+10^{-x_r/a'}}\quad[3]$$

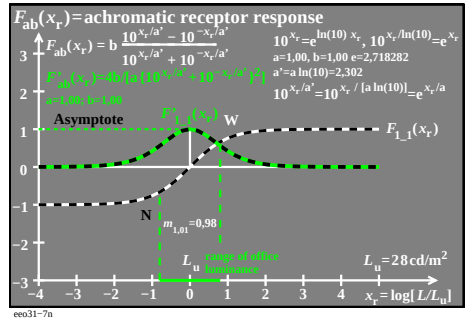
$$\tanh(x/2)=\frac{\sinh(x)}{\cosh(x)+1}=\frac{\cosh(x)+1}{\sinh(x)}=\frac{10^{x_r/2a'}-10^{-x_r/2a'}}{10^{x_r/2a'}+10^{-x_r/2a'}}\quad[4]$$

$$\sinh^2(x)+\cosh^2(x)=1\quad[5]$$

eee30−8n

$F_{ab}(x_r)$=achromatic receptor response

$F_{ab}(x_r)=b\,\frac{10^{x_r/a'}-10^{-x_r/a'}}{10^{x_r/a'}+10^{-x_r/a'}}$ $10^{x_r}=e^{\ln(10)\,x_r}$, $10^{x_r/\ln(10)}=e^{x_r}$
a=1,00, b=1,00 e=2,718282
a'=a ln(10)=2,302
$10^{x_r/a'}=10^{x_r/[a\ln(10)]}=e^{x_r/a}$




eee31−7n

**Mathematical equations of hyperbel functions**

See: *Papula, L., (2003), Mathematische Formelsammlung, Vieweg*

$$F_{ab}(x/a)=b\tanh(x/a)=b\frac{e^{x/a}-e^{-x/a}}{e^{x/a}+e^{-x/a}}=b\frac{u(x/a)}{v(x/a)}\quad[1]$$

$$F'_{ab}(x/a)=b\frac{u'(x/a)\,v(x/a)-u(x/a)\,v'(x/a)}{v^2(x/a)}\quad[2]$$

$$F'_{ab}(x/a)=b\frac{v^2(x/a)-u^2(x/a)}{a\,v^2(x/a)}\quad[3]$$

$$F'_{ab}(x/a)=\frac{4b}{a\,[e^{x/a}+e^{-x/a}]^2}=\frac{b}{a\cosh^2(x/a)}\quad[4]$$

eee31−8n

TUB-test chart eeo3; Model of two normalized response functions $F_{ab}(x_r)$ & $Q_{ab}(x_r)$ and derivation
Tangens hyperbolicus $\tanh(x_r)$ and modified functions with $e^{xr}$ and $10^{xr}$; $a^n=a^{1,0}$