

Weber-Fechner law in CIE 230:2019 for threshold colour differences of surface colours; relations between tristimulus value, lightness and luminance

The *Weber-Fechner* law describes the lightness L_r^* as *logarithmic* function of L_r . The *Stevens* law describes the lightness L_{CIELAB}^* as *potential* function of $L_r=Y/5$. IEC 61966-2-1 uses a similar potential function $L_{\text{IEC}}^* = m L_r^{1/2,4}$.

The *Weber-Fechner* law is equivalent to the equation: $\Delta L_r = c L_r$ [1]

Integration leads to the logarithmic equation: $L_r^* = k \log(L_r)$. [2]

Derivation leads for $\Delta L_r^* = 1$ to the linear equation: $L_r / \Delta L_r = k = 57$. [3]

For *Adjacent* colours in offices the standard contrast range is 25:1=90:3,6.

Table 1: CIE tristimulus value Y , luminance L , and lightness L^*

Colour (matte)	Tristimulus value	office luminance	relative luminance	CIELAB lightness	TUBJND lightness
(contrast) (25:1=90:3,6)	Y	L [cd/m ²]	L_r = L/L_u	L_{CIELAB}^* $\sim m L_r^{1/2,4}$	L_{TUBJND}^* = $k \log(L_r)$
White W (paper)	90 = $18 \cdot 5$	142 = $28,2 \cdot 5$	5	94 = $50 + 44$	40 = $k \log(5)$
Grey Z (paper)	18	28,2	1	50 = 50	0 = $k \log(1)$
Black N (paper)	3,6 = $18/5$	5,6 = $28,2/5$	0,2	18 = $50 - 32$	-40 = $k \log(0,2)$

For the lightness range between $L_r^* = -40$ and 40 the constant is: $k = 40 / \log(5) = 57$