

Line-element examples for grey samples (0.2≤x≤5)

$F(x)$ is called the line-element function of $f(x)$.
 The following relations are valid for $x=Y/Y_0=1/18$:

$$\frac{dF(x)}{dx} = f(x) \quad (1)$$

$$F(x) = \int \frac{f(x)}{f(x_0)} dx \quad (2)$$
 Example for normalized tristimulus value $x=Y/Y_0$:

$$\frac{d[\ln(1+b)x]}{dx} = \frac{b}{1+bx} \quad (3)$$

$$\ln(1+bx) = \int \frac{b}{1+bx} dx \quad (4)$$

DEQ6-19

Line-element equations according to CIE 230:2019

$F_0(x)$ is called the line-element function of $f_0(x)$.
 Both functions are normalized to the surround value:

$$\frac{d[F_0(x)]}{dx} = f_0(x) \quad (1)$$

$$F_0(x) = \int \frac{f_0(x)}{f_0(x_0)} dx = \int \frac{b}{1+bx} dx \quad (2)$$
 Example for $L^*(x)$ and ΔY with $x=Y/Y_0$, $x_0=1$, $b=6,141$:

$$L^*(x) = \frac{L^*(x)}{L^*(x_0)} \frac{\ln(1+bx)}{\ln(1+b)} \quad (3)$$

$$f_0(x) = \frac{\Delta Y}{\Delta Y_0} \frac{1+bx}{1+b} \quad (4)$$

DEQ6-20

Line-element equations according to CIE 230:2019

Colour-discrimination function $f(x) = \Delta Y = \Delta x Y_0$ [0]
 $\Delta Y = (A_1 + A_2 Y) A_0$ $A_0 = 1.5$, $A_1 = 0.0170$, $A_2 = 0.0058$

$$f_0(x) = \frac{\Delta Y}{\Delta Y_0} = \frac{1+bx}{1+b} \quad b = A_2 Y_0 / A_1 \quad x = Y/Y_0 \quad (1)$$

$$F_0(x) = \int \frac{f_0(x)}{f_0(x_0)} dx = \int \frac{b}{1+bx} dx \quad (2)$$
 Example for $L^*(x)$ and ΔY with $x=Y/Y_0$, $x_0=1$, $b=6,141$:

$$L^*(x) = \frac{L^*(x)}{L^*(x_0)} \frac{\ln(1+bx)}{\ln(1+b)} \quad (3)$$

$$f_0(x) = \frac{\Delta Y}{\Delta Y_0} \frac{1+bx}{1+b} \quad (4)$$

DEQ6-21

Line-element equations for thresholds and scaling

Colour-discrimination function $f(x) = \Delta Y = \Delta x Y_0$ [0]
 $\Delta Y = l/(1+x)(2+x) = l/(1+x) - l/(2+x) \quad x = \sqrt{2} e^{(b-1)x}$

$$f_0(x) = \frac{\Delta Y}{\Delta Y_0} = \frac{1+x}{2} - \frac{2+x}{3} \quad x = Y/Y_0 \quad (1)$$

$$F_0(x) = \int \frac{f_0(x)}{f_0(x_0)} dx = \int \frac{1}{1+x} dx - \int \frac{1}{2+x} dx \quad (2)$$
 Example for $L^*(x)$ and ΔY with $x=Y/Y_0$, $x_0=1$:

$$L^*(x) = \frac{L^*(x)}{L^*(x_0)} \frac{\ln(1+x)}{\ln(2)} - \frac{\ln(1+0.5x)}{\ln(1.5)} \quad (3)$$

$$f_0(x) = \frac{\Delta Y}{\Delta Y_0} = \frac{1+x}{2} - \frac{1+0.5x}{1.5} \quad (4)$$

see K. Richter (1985), Computer Graphics and Colorimetry, p. 113–127
<http://color.li.tu-berlin.de/BUA/BPP/PDF>

DEQ6-22

Line-element examples for grey samples (0.2≤x≤5)

$F_0(x)$ is called the line-element function of $f_0(x)$.
 Both functions are normalized to the surround value:

$$\frac{d[F_0(x)]}{dx} = f_0(x) \quad (1)$$

$$F_0(x) = \int \frac{f_0(x)}{f_0(x_0)} dx \quad (2)$$
 Example for the normalized functions with $x_0=1$:

$$F_0(x) = \frac{F(x)}{F(x_0)} \frac{\ln(1+bx)}{\ln(1+b)} \quad (3)$$

$$f_0(x) = \frac{f(x)}{f(x_0)} = \frac{1+bx}{1+b} \quad (4)$$

DEQ6-23

Line-element equations according to CIE 230:2019

Colour-threshold (0) function $f_0(x) = \Delta Y_1 = \Delta x Y_0$ [0]
 $\Delta Y_1 = (A_1 + A_2 Y) A_0$ $A_0 = 1.5$, $A_1 = 0.0170$, $A_2 = 0.0058$

$$f_0(x) = \frac{\Delta Y_1}{\Delta Y_{10}} = \frac{1+bx}{1+b} \quad b = A_2 Y_0 / A_1 \quad x = Y/Y_0 \quad (1)$$

$$F_0(x) = \int \frac{f_0(x)}{f_0(x_0)} dx = \int \frac{b}{1+bx} dx \quad (2)$$
 Example for $L^*(x)$ and ΔY with $x=Y/Y_0$, $x_0=1$, $b=6,141$:

$$L^*(x) = \frac{L^*(x)}{L^*(x_0)} \frac{\ln(1+bx)}{\ln(1+b)} \quad (3)$$

$$f_0(x) = \frac{\Delta Y_1}{\Delta Y_{10}} \frac{1+bx}{1+b} \quad (4)$$

DEQ6-24

Line-element equations for thresholds and scaling

Colour-discrimination function $f(x) = \Delta Y = \Delta x Y_0$ [0]
 $\Delta Y = l/(1+x)(2+x) = l/(1+x) - l/(2+x) \quad x = \sqrt{2} e^{(b-1)x}$

$$f_0(x) = \frac{\Delta Y}{\Delta Y_0} = \frac{1+bx}{1+b} - \frac{1+0.5bx}{1+0.5b} \quad b=1, \quad x = Y/Y_0 \quad (1)$$

$$F_0(x) = \int \frac{f_0(x)}{f_0(x_0)} dx = \int \frac{b}{1+bx} dx - \int \frac{0.5b}{1+0.5bx} dx \quad (2)$$
 Example for $L^*(x)$ and ΔY with $x=Y/Y_0$, $x_0=1$, $b=1$:

$$L^*(x) = \frac{L^*(x)}{L^*(x_0)} \frac{\ln(1+bx)}{\ln(1+b)} - \frac{\ln(1+0.5bx)}{\ln(1+0.5b)} \quad (3)$$

$$f_0(x) = \frac{\Delta Y}{\Delta Y_0} = \frac{1+bx}{1+b} - \frac{1+0.5bx}{1+0.5b} \quad (4)$$

see K. Richter (1985), Computer Graphics and Colorimetry, p. 113–127
<http://color.li.tu-berlin.de/BUA/BPP/PDF>

DEQ6-25

Line-element equations for thresholds and scaling

Colour-discrimination function $f(x) = \Delta Y = \Delta x Y_0$ [0]
 $\Delta Y = l/(1+y)(2+y) = l/(1+y) - l/(2+y) \quad y = 1 + \sqrt{2} e^{(b-1)y}$

$$f_0(y) = \frac{\Delta Y}{\Delta Y_0} = \frac{1+y}{2} - \frac{1+y}{3} \quad y = 1 + Y/Y_0, \quad dy = dx \quad (1)$$

$$F_0(y) = \int \frac{f_0(y)}{f_0(y_0)} dy = \int \frac{1}{1+y} dy - \int \frac{1}{2+y} dy \quad (2)$$
 Example for $L^*(y)$ and ΔY with $y=Y/Y_0+1$:

$$L^*(y) = \frac{L^*(y)}{L^*(y_0)} \frac{\ln(2)}{\ln(3)} - \frac{\ln(1+y)}{\ln(1.5)} \quad (3)$$

$$f_0(y) = \frac{\Delta Y}{\Delta Y_0} = \frac{1+y}{2} - \frac{1+y}{3} \quad (4)$$

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DEQ6-26

Line-element examples for grey samples (0.2≤Y≤5)

$F(Y)$ is called the line-element function of $f(Y)$.
 The following relations are valid for $Y_1/Y_0=1/18$:

$$\frac{dF(Y)}{dY} = f(Y) \quad (1)$$

$$F(Y) = \int \frac{f(Y)}{f(Y_0)} dY \quad (2)$$
 Example for the normalized tristimulus value Y_1/Y_0 :

$$\frac{d[\ln(1+bY)]}{dY} = \frac{b}{1+bY} \quad (3)$$

$$\ln(1+bY) = \int \frac{b}{1+bY} dY \quad (4)$$

DEQ6-27

Line-element examples for grey samples (0.2≤Y≤5)

$F_1(Y)$ is called the line-element function of $f_1(Y)$.
 Both functions are normalized to the surround value:

$$\frac{d[F_1(Y)]}{dY} = f_1(Y) \quad (1)$$

$$F_1(Y) = \int \frac{f_1(Y)}{f_1(Y_0)} dY = \int \frac{b}{1+bY} dY \quad (2)$$
 Example for $L^*(Y)$ and ΔY with $Y_0=1$, $b=6,141$:

$$L^*(Y) = \frac{L^*(Y)}{L^*(Y_0)} \frac{\ln(1+bY)}{\ln(1+b)} \quad (3)$$

$$f_1(Y) = \frac{\Delta Y}{\Delta Y_0} \frac{1+bY}{1+b} \quad (4)$$

DEQ6-28

Line-element equations according to CIE 230:2019

Colour-discrimination function $f(Y) = \Delta Y_1$ [0]
 $\Delta Y_1 = (A_1 + A_2 Y) A_0$ $A_0 = 1.5$, $A_1 = 0.0170$, $A_2 = 0.0058$

$$f_1(Y) = \frac{\Delta Y_1}{\Delta Y_{10}} = \frac{1+bY}{1+b} \quad b = A_2 Y_0 / A_1 \quad Y_1 = Y/Y_0 \quad (1)$$

$$F_1(Y) = \int \frac{f_1(Y)}{f_1(Y_0)} dY = \int \frac{b}{1+bY} dY \quad (2)$$
 Example for $L^*(Y)$ and ΔY with $Y_0=1$, $b=6,141$:

$$L^*(Y) = \frac{L^*(Y)}{L^*(Y_0)} \frac{\ln(1+bY)}{\ln(1+b)} \quad (3)$$

$$f_1(Y) = \frac{\Delta Y_1}{\Delta Y_{10}} \frac{1+bY}{1+b} \quad (4)$$

DEQ6-29

Line-element equations for thresholds and scaling

Colour-discrimination function $f(Y) = \Delta Y_1$, $u_1 = \ln Y$ [0]
 $\Delta Y_1 = l/(1+Y)(2+Y) = l/(1+Y) - l/(2+Y) \quad Y_1 = \sqrt{2} e^{(b-1)Y}$

$$f_1(Y) = \frac{\Delta Y_1}{\Delta Y_{10}} = \frac{1+Y}{2} - \frac{2+Y}{3} \quad Y_1 = Y/Y_0 \quad (1)$$

$$F_1(Y) = \int \frac{f_1(Y)}{f_1(Y_0)} dY = \int \frac{1}{1+Y} dY - \int \frac{1}{2+Y} dY \quad (2)$$
 Example for $L^*(Y)$ and ΔY with $Y_0=1$:

$$L^*(Y) = \frac{L^*(Y)}{L^*(Y_0)} \frac{\ln(1+Y)}{\ln(2)} - \frac{\ln(1+0.5Y)}{\ln(1.5)} \quad (3)$$

$$f_1(Y) = \frac{\Delta Y_1}{\Delta Y_{10}} = \frac{1+Y}{2} - \frac{1+0.5Y}{1.5} \quad (4)$$

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DEQ6-30

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 Both functions are normalized to the surround value:

$$\frac{d[F_0(Y)]}{dY} = f_0(Y) \quad (1)$$

$$F_0(Y) = \int \frac{f_0(Y)}{f_0(Y_0)} dY \quad (2)$$
 Example for the normalized functions with $Y_0=1$:

$$F_0(Y) = \frac{F(Y)}{F(Y_0)} \frac{\ln(1+bY)}{\ln(1+b)} \quad (3)$$

$$f_0(Y) = \frac{f(Y)}{f(Y_0)} = \frac{1+bY}{1+b} \quad (4)$$

DEQ6-31

Line-element equations according to CIE 230:2019

Colour-threshold (0) function $f_0(Y) = \Delta Y_1 = \Delta Y Y_0$ [0]
 $\Delta Y_1 = (A_1 + A_2 Y) A_0$ $A_0 = 1.5$, $A_1 = 0.0170$, $A_2 = 0.0058$

$$f_0(Y) = \frac{\Delta Y_1}{\Delta Y_{10}} = \frac{1+bY}{1+b} \quad b = A_2 Y_0 / A_1 \quad Y_1 = Y/Y_0 \quad (1)$$

$$F_0(Y) = \int \frac{f_0(Y)}{f_0(Y_0)} dY = \int \frac{b}{1+bY} dY \quad (2)$$
 Example for $L^*(Y)$ and ΔY with $Y_0=1$, $b=6,141$:

$$L^*(Y) = \frac{L^*(Y)}{L^*(Y_0)} \frac{\ln(1+bY)}{\ln(1+b)} \quad (3)$$

$$f_0(Y) = \frac{\Delta Y_1}{\Delta Y_{10}} \frac{1+bY}{1+b} \quad (4)$$

DEQ6-32

Line-element equations for thresholds and scaling

Colour-discrimination function $f(Y) = \Delta Y_1$, $u_1 = \ln Y$ [0]
 $\Delta Y_1 = l/(1+Y)(2+Y) = l/(1+Y) - l/(2+Y) \quad Y_1 = \sqrt{2} e^{(b-1)Y}$

$$f_1(Y) = \frac{\Delta Y_1}{\Delta Y_{10}} = \frac{1+bY}{1+b} - \frac{1+0.5bY}{1+0.5b} \quad b=1, \quad Y_1 = Y/Y_0 \quad (1)$$

$$F_1(Y) = \int \frac{f_1(Y)}{f_1(Y_0)} dY = \int \frac{b}{1+bY} dY - \int \frac{0.5b}{1+0.5bY} dY \quad (2)$$
 Example for $L^*(Y)$ and ΔY with $Y_0=1$, $b=1$:

$$L^*(Y) = \frac{L^*(Y)}{L^*(Y_0)} \frac{\ln(1+bY)}{\ln(1+b)} - \frac{\ln(1+0.5bY)}{\ln(1+0.5b)} \quad (3)$$

$$f_1(Y) = \frac{\Delta Y_1}{\Delta Y_{10}} \frac{1+bY}{1+b} - \frac{1+0.5bY}{1+0.5b} \quad (4)$$

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DEQ6-33

Line-element equations for thresholds and scaling

Colour-discrimination function $f(Y) = \Delta Y = \Delta Y Y_0$ [0]
 $\Delta Y = l/(1+y)(2+y) = l/(1+y) - l/(2+y) \quad y = 1 + \sqrt{2} e^{(b-1)y}$, $u_1 = \ln Y$

$$f_0(y) = \frac{\Delta Y}{\Delta Y_0} = \frac{1+y}{2} - \frac{1+y}{3} \quad y = 1 + Y/Y_0, \quad dy = dx \quad (1)$$

$$F_0(y) = \int \frac{f_0(y)}{f_0(y_0)} dy = \int \frac{1}{1+y} dy - \int \frac{1}{2+y} dy \quad (2)$$
 Example for $L^*(y)$ and ΔY with $y=Y/Y_0+1$:

$$L^*(y) = \frac{L^*(y)}{L^*(y_0)} \frac{\ln(2)}{\ln(3)} - \frac{\ln(1+y)}{\ln(1.5)} \quad (3)$$

$$f_0(y) = \frac{\Delta Y}{\Delta Y_0} = \frac{1+y}{2} - \frac{1+y}{3} \quad (4)$$

see K. Richter (1985), Computer Graphics and Colorimetry, p. 113–127
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