

LABJND modifications of the colour space and the colour-difference formula by line elements for different applications

Example line element in lightness direction of *Stiles* (1946), who uses the luminance L instead of the tristimulus value Y .

$$L^*(Y) = s \ln[1+9Y] \quad s = \text{scaling factor} \quad [1a]$$

$$d(L^*(Y)) / dY = 9s / [1+9Y] \quad [2a]$$

For this derivation, and for the lightness threshold $d(L^*(Y))=1$ it is valid:

$$dY = [1+9Y] / 9s \quad [3a]$$

For the normalization with the surround values $Y_u=18$, dY_u and $L^*(Y_u)$:

$$dY/dY_u = [1+9Y] / [1+9Y_u] \quad [4a]$$

$$L^*(Y) / L^*(Y_u) = \ln[1+9Y] / \ln[1+9Y_u] \quad [5a]$$

For the LABJND colour-difference formula according to CIE 230:2019:

$$dY/dY_u = [A_1 + A_2 Y] / [A_1 + A_2 Y_u] \quad A_1 = 0,0170, A_2 = 0,0058 \quad [1b]$$

It is valid with the definition $A_{20} = A_2 Y_u / A_1 = 6,141$

$$dY/dY_u = [1 + A_{20}(Y/Y_u)] / [1 + A_{20}] \quad [2b]$$

$$L^*(Y) / L^*(Y_u) = \ln[1 + A_{20}(Y/Y_u)] / \ln[1 + A_{20}] \quad [3b]$$

Line elements for the LABJND-colour space are NOT included in CIE 230.

The value of A_{20} is known for many applications, for example as function of the distance, the presentation time and the luminance of the samples.

CIELAB modifications of the colour space and the colour-difference formula by line elements for different applications

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For the CIELAB colour-space according to ISO/CIE 11664-4:2019

$$L^*(Y) = s [Y/Y_u]^k - 16 = s_s [Y/Y_u]^k - 16 \quad s = 116, k = 1/3, Y_u = 100, Y_u = 18 \quad [1b]$$

$$d(L^*(Y)) / dY = s_s k [Y/Y_u]^{k-1} \quad \text{with } s_s = 116 [Y_u/Y_u]^k = 65,50, 1 < Y < 100 \quad [2b]$$

For this derivation, and for the lightness threshold $d(L^*(Y))=1$ it is valid:

$$dY = [Y/Y_u]^{1-k} / s_s k \quad [3b]$$

$$dY/dY_u = [Y/Y_u]^{1-k} \quad [4b]$$

$$L^*(Y) / L^*(Y_u) = \{s_s [Y/Y_u]^k - 16\} / \{s_s - 16\} \quad [5b]$$

Line elements for the CIELAB-colour space are included in ISO/CIE 11664-4.

Colour-difference formula LABJND 1985 (JND=just noticeable difference)

$$\Delta E_{JND}^* = \Delta E_{85}^* = A_0 [(\Delta Y)^2 + (A_3 \Delta a^*)^2 + (A_4 \Delta b^*)^2]^{1/2} / (A_1 + A_2 \cdot Y) \quad [1]$$

$$a^* = x/y \quad a_n = x_n/y_n \quad b^* = -0,4 z/Y \quad b_n = -0,4 z_n/y_n \quad [2]$$

$$a'' = a_n + (a - a_n) / (1 + 0,5 |a - a_n|) \quad n = D65 \text{ or } A \text{ (background)} \quad [3]$$

$$b'' = b_n + (b - b_n) / (1 + 0,5 |b - b_n|) \quad [4]$$

$$Y = (Y_1 + Y_2) / 2 \quad \Delta Y = Y_1 - Y_2 \quad \Delta a'' = a_1'' - a_2'' \quad \Delta b'' = b_1'' - b_2'' \quad [5]$$

$$A_1 = 0,0170 \quad A_2 = 0,0058 \quad [6]$$

$$A_3 = 1,0 \quad A_4 = 1,8 \quad A_0 = 1,5 \quad \text{background D65} \quad [7]$$

$$A_3 = 1,0 \quad A_4 = 1,7 \quad A_0 = 1,0 \quad \text{background A} \quad [8]$$

Just noticeable difference (JND) in four colour directions

$$\Delta Y = \text{const} (A_1 + A_2 \cdot Y) / A_0 \quad \text{in luminance direction } WN \quad [1a]$$

$$\Delta a'' \cdot Y = \text{const} (A_1 + A_2 \cdot Y) / (A_0 \cdot A_3) \quad \text{in chromaticity direction } RG \quad [2a]$$

$$\Delta b'' \cdot Y = \text{const} (A_1 + A_2 \cdot Y) / (A_0 \cdot A_4) \quad \text{in chromaticity direction } YB \quad [3a]$$

$$\Delta c_{ab}'' \cdot Y = \text{const} (A_1 + A_2 \cdot Y) / (A_0 \cdot [A_3^2 + A_4^2]^{1/2}) \quad \text{in chromaticity direction } c_{ab} \quad [4a]$$

Colour-difference formula LABJND 1985 for near achromatic colours

$$\Delta E_{JND}^* = \Delta E_{85}^* = A_0 [(\Delta Y)^2 + (A_3 \Delta a^*)^2 + (A_4 \Delta b^*)^2]^{1/2} / (A_1 + A_2 \cdot Y) \quad [1]$$

$$a^* = x/y \quad b^* = -0,4 z/y \quad [2]$$

$$Y = (Y_1 + Y_2) / 2 \quad \Delta Y = Y_1 - Y_2 \quad \Delta a = a_1 - a_2 \quad \Delta b = b_1 - b_2 \quad [3]$$

$$A_1 = 0,0170 \quad A_2 = 0,0058 \quad [4]$$

$$A_3 = 1,0 \quad A_4 = 1,8 \quad A_0 = 1,5 \quad \text{background D65} \quad [5]$$

$$A_3 = 1,0 \quad A_4 = 1,7 \quad A_0 = 1,0 \quad \text{background A} \quad [6]$$

Just noticeable difference (JND) in three colour directions and line elements

$$A_0 \cdot \Delta Y = (A_1 + A_2 \cdot Y) \quad \text{in luminance direction } WN \quad [1a]$$

$$A_0 \cdot \Delta a \cdot A_3 \cdot Y = (A_1 + A_2 \cdot Y) \quad \text{in chromaticity direction } RG \quad [2a]$$

$$A_0 \cdot \Delta b \cdot A_4 \cdot Y = (A_1 + A_2 \cdot Y) \quad \text{in chromaticity direction } YB \quad [3a]$$

$$dE_{85,L}^* = \frac{\delta}{\delta Y} L_{85}^* = \frac{\delta}{\delta Y} [(A_0 / A_2) \cdot \ln(A_1 + A_2 \cdot Y)] = A_0 \cdot dY / (A_1 + A_2 \cdot Y) \quad [4a]$$

$$dE_{85,a}^* = \frac{\delta}{\delta a} a_{85}^* = \frac{\delta}{\delta a} [(A_0 \cdot A_3 \cdot Y \cdot a) / (A_1 + A_2 \cdot Y)] = A_0 \cdot da \cdot A_3 \cdot Y / (A_1 + A_2 \cdot Y) \quad [5a]$$

$$dE_{85,b}^* = \frac{\delta}{\delta b} b_{85}^* = \frac{\delta}{\delta b} [(A_0 \cdot A_4 \cdot Y \cdot b) / (A_1 + A_2 \cdot Y)] = A_0 \cdot db \cdot A_4 \cdot Y / (A_1 + A_2 \cdot Y) \quad [6a]$$