

**Weber-Fechner law in CIE 230:2019 for threshold colour differences of surface colours**

The Weber-Fechner law describes the lightness  $L^*$ , as logarithmic function of  $L_u$ .  
 The Stevens law describes the lightness  $L^*_{REL,LAB}$  as potential function of  $L_u = 3/5 L^*$ .  
 IEC 61966-2-1 uses a similar potential function  $L^*_{TIC} = m L_u^{1/2.4}$ .

The Weber-Fechner law is equivalent to the equation:  $\Delta L_u = c L_u$  [1]  
 Integration leads to the logarithmic equation:  $L^* = k \log(L_u)$  [2]  
 Derivation for  $\Delta L_u = 1$  leads to the linear equation:  $L_u / \Delta L_u = k_1$  ( $k_0=46, k_1=63$ ) [3]  
 For colours in offices the standard contrast range is 25:1=90:3.6

**Table 1: CIE tristimulus value Y, luminance  $L_u$ , and lightness  $L^*$**

Colour (matte)	Tristimulus value Y	office luminance $L_u$ [cd/m <sup>2</sup> ]	relative luminance $L_u/L_u$	CIE lightness $L^*$	relative lightness $L^*/L^*$
White W (paper)	90	142	5	94	44
Grey Z (paper)	18	28.2	1	50	40
Black N (paper)	3.6	5.6	0.2	18	32

For the lightness range between  $L^* = -40$  and 40 the constant is:  $k=40 \log(5) = 57$

**Weber-Fechner law in CIE 230:2019 for threshold colour differences of surface colours and two ranges  $0.2 \leq L_u \leq 1$  and  $1 \leq L_u \leq 5$**

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The Weber-Fechner law is equivalent to the linear equation:  $\Delta L_u = c L_u$  ( $c=0.1$ ) [1]  
 Integration leads to the logarithmic equation:  $L^* = k_1 \log(L_u)$  ( $k_1=63$ ) [2]  
 Derivation leads for  $L^* = 1$  to the linear equation:  $L_u / \Delta L_u = k_1$  ( $k_0=46, k_1=63$ ) [3]  
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For the two lightness ranges it is  $k_0 = -32 \log(0.2) = 46$  and  $k_1 = 44 \log(5) = 63$ .

**line element of Stiles (1946) with „color values“  $L_P, M_D, S_T$**

**three separate color signal functions**

$$F(L_P) = i \ln(1 + 9 L_P)$$

$$F(M_D) = j \ln(1 + 9 M_D)$$

$$F(S_T) = k \ln(1 + 9 S_T)$$

**Taylor-derivations:**

$$\Delta F(L_P, M_D, S_T) = \frac{dF}{dL_P} \Delta L_P + \frac{dF}{dM_D} \Delta M_D + \frac{dF}{dS_T} \Delta S_T$$

$$= \frac{9i}{1+9L_P} \Delta L_P + \frac{9j}{1+9M_D} \Delta M_D + \frac{9k}{1+9S_T} \Delta S_T$$

**line element of Vos&Walraven (1972) with „color values“  $L_P, M_D, S_T$**

**three separate color signal functions**

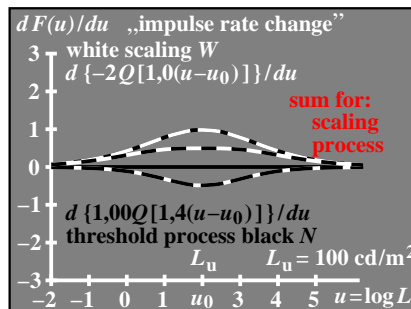
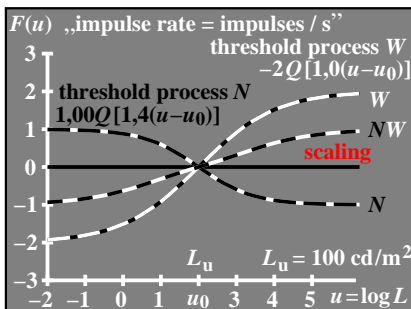
$$F(L_P) = -2i \sqrt{L_P}$$

$$F(M_D) = -2j \sqrt{M_D}$$

$$F(S_T) = -2k \sqrt{S_T}$$

**Taylor-derivations:**

$$\Delta F(L_P, M_D, S_T) = \frac{dF}{dL_P} \Delta L_P + \frac{dF}{dM_D} \Delta M_D + \frac{dF}{dS_T} \Delta S_T$$

$$\Delta F(L_P, M_D, S_T) = \frac{i}{\sqrt{L_P}} \Delta L_P + \frac{j}{\sqrt{M_D}} \Delta M_D + \frac{k}{\sqrt{S_T}} \Delta S_T$$


**functions  $q[k(u-u_0)]$**

**„achromatic signal“-description**

with  $u = \log L$  ( $L = \text{luminance}$ )  
 $u_0 = \log L_u$  ( $L_u = \text{surround luminance}$ )

$$q[k(u-u_0)] = 1 + 1/[1 + \sqrt{2} e^{k(u-u_0)}]$$

**function values:**

$$q[k(u-u_0) \rightarrow +\infty] = 1$$

$$q[k(u-u_0) = 0] = \sqrt{2}$$

$$q[k(u-u_0) \rightarrow -\infty] = 2$$

**„achromatic signal“-description**

**functions  $Q_{1m}[k(u-u_0)]$**

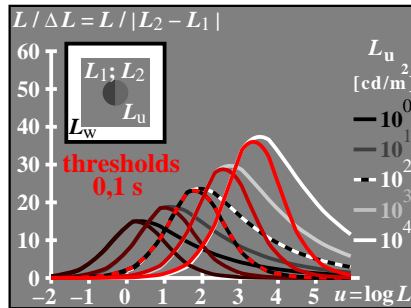
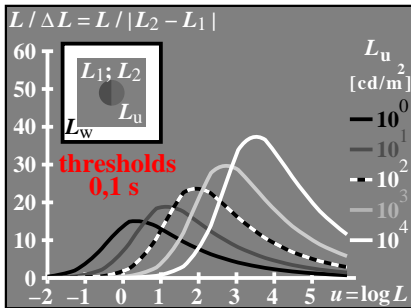
with  $u = \log L$  ( $L = \text{luminance}$ )  
 $u_0 = \log L_u$  ( $L_u = \text{surround luminance}$ )

$$Q_{1m}[k(u-u_0)] = \frac{l}{\ln \sqrt{2}} \ln q[k(u-u_0)] - m$$

**function values with  $l = m = 1$ :**

$$Q[k(u-u_0) \rightarrow +\infty] = -1$$

$$Q[k(u-u_0) = 0] = 0$$

$$Q[k(u-u_0) \rightarrow -\infty] = 1$$


**„achromatic signal“ discrimination**

as function of relative light density  
 $h = \ln H = k(u-u_0)$ ,  $\ln = \text{natural log.}$

$$Q' = \frac{d}{dH} [\ln\{1 + 1/(1 + \sqrt{2}H)\}] / \ln \sqrt{2}$$

$$= -\sqrt{2} / [\ln \sqrt{2} (1 + \sqrt{2}H)(2 + \sqrt{2}H)]$$

**function values:**

$$Q'[k(u-u_0) \rightarrow +\infty] = 0$$

$$Q'[k(u-u_0) = 0] = -0,5$$

$$Q'[k(u-u_0) \rightarrow -\infty] = 0$$

**luminance discrimination**

possibility  $L/\Delta L$  as function of  $H$

with:  $L = 10^u$   $H = e^h = 10^{\log e k(u-u_0)}$

$$dL/du = \ln 10 L$$

$$dH/du = k H$$

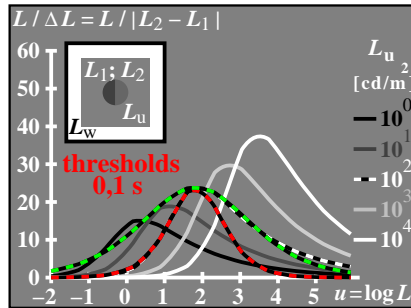
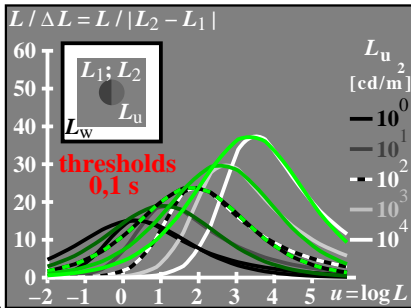
**it follows:  $L/\Delta L = [kH / (dH \ln 10)]$**

$$\frac{L}{\Delta L} = \text{const } H / [(1 + \sqrt{2}H)(2 + \sqrt{2}H)]$$

**function values:**

$$Q'[k(u-u_0) \rightarrow +\infty] = 0$$

$$Q'[k(u-u_0) = 0] = \text{maximum}$$

$$Q'[k(u-u_0) \rightarrow -\infty] = 0$$


**double line element of Richter (1987) for the lighting technology with the luminance  $L = f(L_P, M_D, S_T)$**

$$F(L) = \int_{-\infty}^L (L/\Delta L) dL \quad (\text{relative } L, M, S?)$$

$$F(L) = iQ(H) = \begin{cases} iQ(\bar{H}) & (u < u_0) \\ iQ(\bar{H}) & (u \geq u_0) \end{cases}$$

with:  $k=1,4$   $\bar{k}=1$   $i=1$   $\bar{i}=-2$   
 $u = \log L$   $u_0 = \log L_u$   
 $H = e^{k(u-u_0)}$   $\bar{H} = e^{\bar{k}(u-u_0)}$

**double line element of Richter (1987) for the lighting technology with the luminance  $L = f(L_P, M_D, S_T)$**

$$F(L) = \int_{-\infty}^L (L/\Delta L) dL \quad (\text{relative } L, M, S?)$$

$$F(L) = iQ(H) \quad H = e^{k(u-u_0)}$$

$$Q(H) = [\ln\{1 + 1/(1 + \sqrt{2}H)\}] / \ln \sqrt{2} - 1$$

**Taylor-derivations:**

$$\Delta F(L) = \frac{dF}{dL} \Delta L = i \frac{dQ}{dH} \Delta H$$

$$= -i \sqrt{2} \Delta H / [\ln \sqrt{2} (1 + \sqrt{2}H)(2 + \sqrt{2}H)]$$