LABJND modifications of the colour space and the colour-difference formula by line elements for different applications

Example line element in lightness direction of *Stiles (1946)*, who uses the luminance L instead of the tristimulus value Y.

$L^*(Y) = s \ln[1+9Y]$ s=scaling factor [1a]

$d(L^*(Y)) / dY = 9s / [1+9Y]$ [2a]

For this derivation, and for the lightness threshold $d(L^*(Y))=1$ it is valid:

$dY = [1+9Y] / 9s$ [3a]

For the normalization with the surround values $Y_u=18$, dY_u and $L^*(Y_u)$:

$dY/dY_u = [1+9Y] / [1+9Y_u]$ [4a]

$L^*(Y) / L^*(Y_u) = \ln[1+9Y] / \ln[1+9Y_u]$ [5a]

For the LABJND colour-difference formula according to CIE 230:2019:

$dY/dY_u = [A_1+A_2Y] / [A_1+A_2Y_u]$ $A_1=0{,}0170$, $A_2=0{,}0058$ [1b]

It is valid with the definition $A_{2u}=A_2Y_u/A_1=6{,}141$

$dY/dY_u = [1+A_{2u}(Y/Y_u)] / [1+A_{2u}]$ [2b]

$L^*(Y) / L^*(Y_u) = \ln[1+A_{2u}(Y/Y_u)] / \ln[1+A_{2u}]$ [3b]

Line elements for the LABJND-colour space are NOT included in CIE 230. The value of A_{2u} is known for many applications, for example as function of the distance, the presentation time and the luminance of the samples.

CEA20−3N

CIELAB modifications of the colour space and the colour-difference formula by line elements for different applications

Example line element in lightness direction of *Stiles (1946)*, who uses the luminance L instead of the tristimulus value Y.

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$d(L^*(Y)) / dY = 9s / [1+9Y]$ [2a]

For this derivation, and for the lightness threshold $d(L^*(Y))=1$ it is valid:

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For the normalization with the surround values $Y_u=18$, dY_u and $L^*(Y_u)$:

$dY/dY_u = [1+9Y] / [1+9Y_u]$ [4a]

$L^*(Y) / L^*(Y_u) = \ln[1+9Y] / \ln[1+9Y_u]$ [5a]

For the CIELAB colour-space according to ISO/CIE 11664-4:2019

$L^*(Y) = s\,[Y/Y_n]^k - 16 = s_u[Y/Y_u]^k - 16$ s=116, k=1/3, $Y_n=100$, $Y_u=18$ [1b]

$d(L^*(Y)) / dY = s_u k\,[Y/Y_n]^{k-1}$ with $s_u=116[Y_u/Y_n]^k=65{,}50$, $1<=Y<=100$ [2b]

For this derivation, and for the lightness threshold $d(L^*(Y))=1$ it is valid:

$dY = [Y/Y_u]^{1-k} / s_u k$ [3b]

$dY/dY_u = [Y/Y_u]^{1-k}$ [4b]

$L^*(Y) / L^*(Y_u) = \{s_u[Y/Y_u]^k - 16\} / \{s_u - 16\}$ [5b]

Line elements for the CIELAB-colour space are included in ISO/CIE 11664-4.

CEA20−7N

Colour-difference formula LABJND 1985 (JND=just noticeable difference)

$\Delta E^*_{JND} = \Delta E^*_{85} = A_0\,[(\Delta Y)^2 + (A_3\,\Delta a''\cdot Y)^2 + (A_4\,\Delta b''\cdot Y)^2]^{1/2} / (A_1 + A_2\cdot Y)$ [1]

$a = x / y$ $a_n = x_n / y_n$ $b = -0{,}4\,z / Y$ $b_n = -0{,}4\,z_n / y_n$ [2]

$a'' = a_n + (a - a_n) / (1 + 0{,}5\,|a - a_n|)$ n = D65 or A (background) [3]

$b'' = b_n + (b - b_n) / (1 + 0{,}5\,|b - b_n|)$ [4]

$Y = (Y_1 + Y_2) / 2$ $\Delta Y = Y_1 - Y_2$ $\Delta a'' = a''_1 - a''_2$ $\Delta b'' = b''_1 - b''_2$ [5]

$A_1 = 0{,}0170$ $A_2 = 0{,}0058$ [6]

$A_3 = 1{,}0$ $A_4 = 1{,}8$ $A_0 = 1{,}5$ background D65 [7]

$A_3 = 1{,}0$ $A_4 = 1{,}7$ $A_0 = 1{,}0$ background A [8]

Just noticeable difference (JND) in four colour directions

$\Delta Y = \text{const}\,(A_1 + A_2\cdot Y) / A_0$ in luminance direction *WN* [1a]

$\Delta a''\cdot Y = \text{const}\,(A_1 + A_2\cdot Y) / (A_0\cdot A_3)$ in chromaticity direction *RG* [2a]

$\Delta b''\cdot Y = \text{const}\,(A_1 + A_2\cdot Y) / (A_0\cdot A_4)$ in chromaticity direction *YB* [3a]

$\Delta c_{ab}''\cdot Y = \text{const}\,(A_1+A_2\cdot Y)/(A_0\cdot[A_3^2+A_4^2]^{1/2})$ in chromaticity direction c_{ab} [4a]

CEA21−3N

Colour-difference formula LABJND 1985 for near achromatic colours

$\Delta E^*_{JND} = \Delta E^*_{85} = A_0\,[(\Delta Y)^2 + (A_3\,\Delta a\cdot Y)^2 + (A_4\,\Delta b\cdot Y)^2]^{1/2} / (A_1 + A_2\cdot Y)$ [1]

$a = x / y$ $b = -0{,}4\,z / y$ [2]

$Y = (Y_1 + Y_2) / 2$ $\Delta Y = Y_1 - Y_2$ $\Delta a = a_1 - a_2$ $\Delta b = b_1 - b_2$ [3]

$A_1 = 0{,}0170$ $A_2 = 0{,}0058$ [4]

$A_3 = 1{,}0$ $A_4 = 1{,}8$ $A_0 = 1{,}5$ background D65 [5]

$A_3 = 1{,}0$ $A_4 = 1{,}7$ $A_0 = 1{,}0$ background A [6]

Just noticeable difference (JND) in three colour directions and line elements

$A_0\cdot\Delta Y = (A_1 + A_2\cdot Y)$ in luminance direction *WN* [1a]

$A_0\cdot\Delta a\cdot A_3\cdot Y = (A_1 + A_2\cdot Y)$ in chromaticity direction *RG* [2a]

$A_0\cdot\Delta b\cdot A_4\cdot Y = (A_1 + A_2\cdot Y)$ in chromaticity direction *YB* [3a]

$dE^*_{85,L} = \frac{\delta}{\delta Y} L^*_{85} = \frac{\delta}{\delta Y}[(A_0 / A_2)\cdot ln\,(A_1+A_2\cdot Y)] = A_0\cdot dY/(A_1+A_2\cdot Y)$ [4a]

$dE^*_{85,a} = \frac{\delta}{\delta a} a^*_{85} = \frac{\delta}{\delta a}[(A_0\cdot A_3\cdot Y\cdot a)/(A_1+A_2\cdot Y)] = A_0\cdot da\cdot A_3\cdot Y/(A_1+A_2\cdot Y)$ [5a]

$dE^*_{85,b} = \frac{\delta}{\delta b} b^*_{85} = \frac{\delta}{\delta b}[(A_0\cdot A_4\cdot Y\cdot b)/(A_1+A_2\cdot Y)] = A_0\cdot db\cdot A_4\cdot Y/(A_1+A_2\cdot Y)$ [6a]

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