***Relative* CIELAB data *nce** and *rgb**$_3$ based on eight CIELAB reference colours**

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Remark: This is a proposal for CIE Division 1 “Colour” for discussion at technical meetings. For this report (15 pages, 400 kByte, Version 2007-08-15) see the URL

http://www.ps.bam.de/CIE07R.PDF

Scope

In the CIELAB colour space, see CIE 15, there are two sets of equivalent *absolute* CIELAB coordinates, the cylindric coordinates *L**, *C**$_{ab}$ and *h*$_{ab}$ *(LAB*LCH* = LCH*, described here by capital letters)* and the CIELAB cartesian coordinates *L*, a** and *b* (LAB*LAB* = LAB*).*

This report defines *relative* CIELAB coordinates *lab* (small letters)*. Two new sets of equivalent *relative* CIELAB coordinates *lab*nce* = nce** and *lab*rgb**$_3$ = *rgb**$_3$ are defined. This report defines a linear relation between *relative* CIELAB coordinates *(lab*)* and the absolute CIELAB coordinates *(LAB*)* if eight reference colours in the CIELAB space are given.

In the standard case of this report the *relative* coordinates *nce** and *rgb**$_3$ are calculated in relation to the six chromatic colours with the *LCH** data (*L** = 50, *C**$_{ab}$ = 100, *h*$_{ab}$ = 30, 90, 150, 210, 270, 330) and the two achromatic colours with the *LC** data (*L** = 0 and 100, *C**$_{ab}$ = 0) for Black *N* and White *W*. The system with the above eight reference colours is called Standard Reflective System SRS00. The eight CIELAB reference colours play a unique role in colour vision and therefore the calculation of the *relative* coordinates *nce** and *rgb**$_3$ in relation to these reference colours is appropriate.

In this report three modifications of these reference colours are discussed according to different viewing situations and experimental results.

Remark: The coordinates *nce** of this report are similar compared to the coordinates *relative* blackness, *relative* chroma and elementary hue number used in the *NCS*-colour system (see Svensk standard series SS 01 91 00 to 03:1982)

1. Introduction

In the CIELAB colour space for example the colours of offset printing or the colours on a monitor may be located on a hue triangle as shown in Fig. 1 for a red hue.

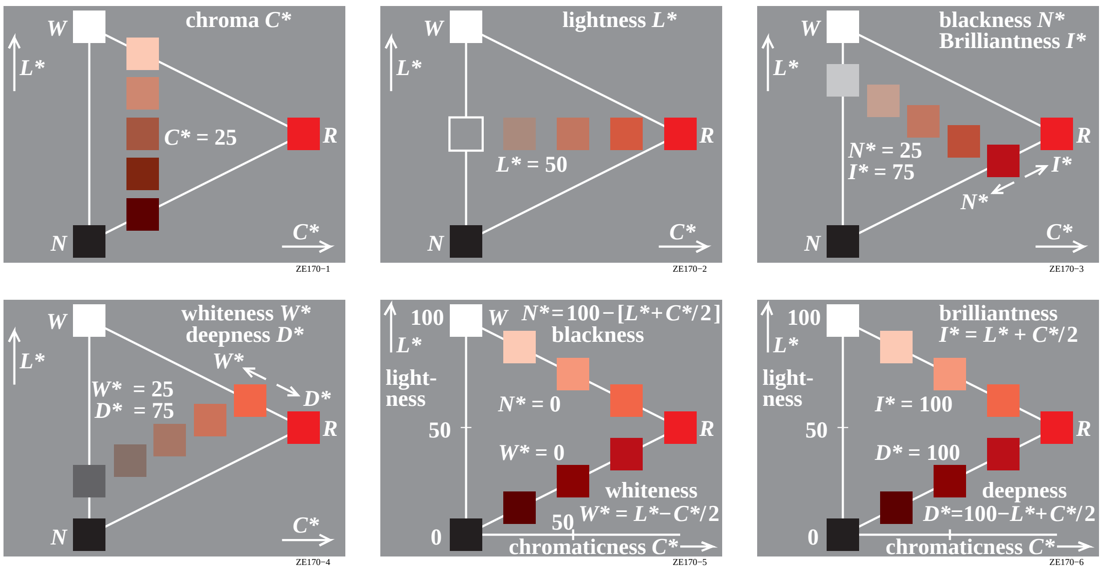

Figure 1: Color attributes *N*, I*, W and *D** in comparison to chroma *C** and lightness *L***

Fig 1 shows colour attributes blackness N^*, brilliantness I^*, whiteness W^* and colour deepness D^* in comparison to chroma C^* and lightness L^*. The colour attributes blackness N^* and brilliantness I^* as well as whiteness W^* and colour deepness D^* are complementary. Therefore it is valid

$$N^* = 100 - I^* \tag{1}$$

$$W^* = 100 - D^* \tag{2}$$

Further there is the hue triangle equation of*Ostwald* (1930)

$$N^* + W^* + C^* = 100 \tag{3}$$

Additionally there are linear relations between the four coordinates N^*, W^*, L^* and C^*

$$N^* = 100 - [\, L^* + 0{,}5\, C^* \,] \tag{4}$$

$$W^* = L^* - 0{,}5\, C^* \tag{5}$$

All colorimetric colour systems use three colour attributes for the specifications of colours. Most colour systems use the hue as the first and most important colour attribute and distinguish in the choice of the two others. In the CIELAB system the attributes chroma C^*_{ab} and lightness L^* are preferred. In the Swedish Natural Colour Systems *NCS* instead of the lightness L^* the blackness N^* is used.

According to the experimental results of *Evans* (1974) the colours of equal blackness N^* are located in the CIELAB colour space on a line independent of hue described by

$$N^* = 100 - [\, L^* + 0{,}5\, C^*_{ab}] \qquad (L^*,\, C^*_{ab} \text{ of CIELAB}) \tag{6}$$

Remark: This corresponds to $N^* = 10\ \{ 10 - [V + C]\}$ with V = Value and C = Chroma of the Munsell system which has been studied by *Evans* (1974).

In many cases the colour difference of the 5 steps between black *N* and white *W* and the 5 steps between the six chromatic colours *X=RJGC'BM'* and both Black *N* and White *W* should be equally spaced in CIELAB, compare Fig. 1.

For many example files with 5 and 16 step colour scales in up to 10 hue planes see the URL

http://www.ps.bam.de/RLAB00E

In applications observers often evaluate the colours *relative* to a given media Black *N* and White *W* and a most chromatic colour an observer has in mind. In this case *relative* colour attributes, for example *relative* blackness n^*, *relative* lightness l^*, *relative* chroma c^* and other may be appropriate.

Therefore in applications instead of the *absolute* data N^*, I^*, W^*, D^*, L^* and C^* *(capital letters)* often the *relative* data n^*, i^*, w^*, d^*, l^* and c^* *(small letters)* are used.

The *relative* colour attributes, for example *relative* blackness n^* and *relative* chroma c^*, and the linear relations to the CIELAB data *LCH** are described in this technical report.

A description of many other visual attributes which may be described by the coordinates of this report, for example absolute and chromatic thresholds, colour differences at threshold for adjacent and separate colour samples in a gray background is beyond the scope of this report, see for example *Richter* (2006a, b, c)

2. Three modifications of the reference system SRS00 for applications

There are three modification of the reference system SRS00 with the CIELAB *LCH** data (L^* = 50, C^*_{ab} = 100, h_{ab} = 30, 90, 150, 210, 270, 330). Six modified hue angles 26, 92, 162, 217, 272 and 329 seem to be more appropriate according to different experimental results for the elementary hues. This system is called the Natural Reflective System NRS00. The value 00 is used for the lightness $L^*_N = 0$ for black *N*. In ISO/IEC 15775 for practical applications the lightness L^*_N = 18,01 and L^*_W = 95.41 is used as reference for media Black *N* and White *W*. In the reference system SRS00 the chroma is equal to the lightness difference between White *W* and Black *N*. Therefore in the application case according to ISO/IEC 15775 the CIELAB chroma C^*_{ab} reduces from 100 to 77,4.

As a result three modifications of the reference system SRS00 will be studied. SRS18 with a reduced chroma and NRS100 and NRS18 both with changed elementary hues compared to SRS00 and SRS18.

We will study both *absolute* CIELAB data *LCH** = *LAB*LCH** and *relative* CIELAB data *nce** = *lab*nce** and rgb^*_3 = $lab^*rgb^*_3$.

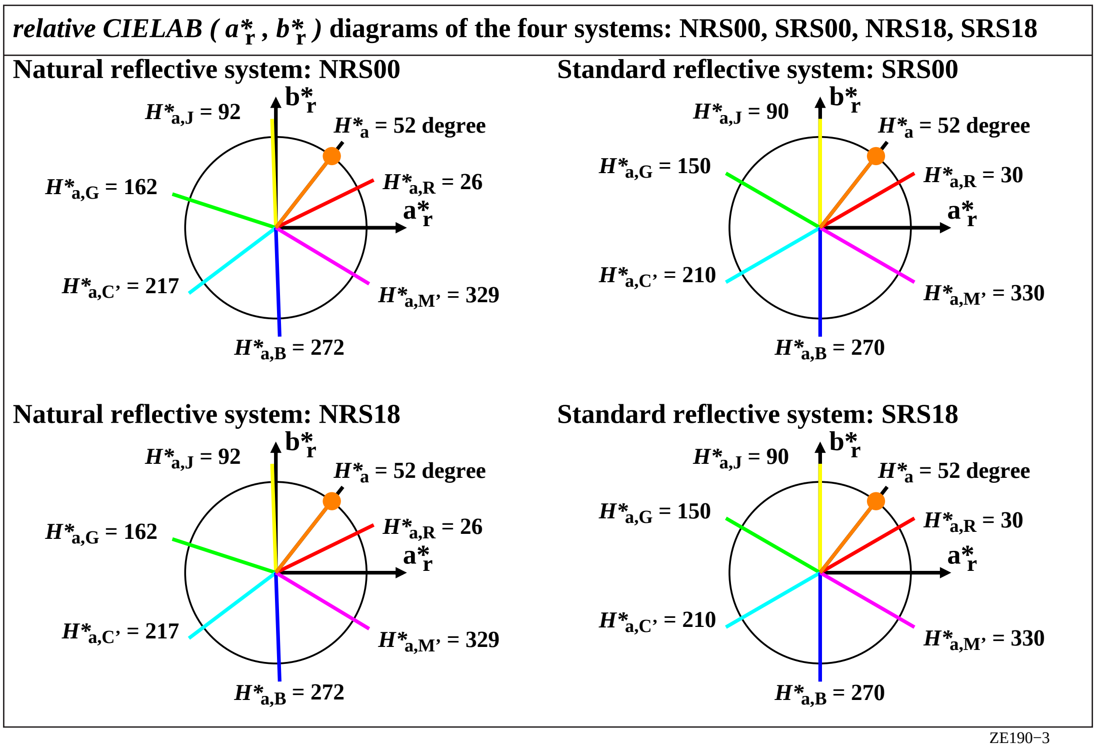

Figure 2: Relative CIELAB (a^*_r, b^*_r) diagram of four systems

Figure 2 shows the relative CIELAB (a^*_r, b^*_r) diagram of four systems NRS00, SRS00, NRS18 and SRS18. In Fig. 2 both the systems NRS00 and NRS18 as well as SRS00 and SRS18 are equal. The systems SRS00 and SRS18 use the standard 60-degree hue angle difference and the systems NRS00 and NRS18 the elementary hue angles of the CIE-test colours no. 9 to 12 of CIE 13.3 (Colour Rendering) for CIE illuminant D65.

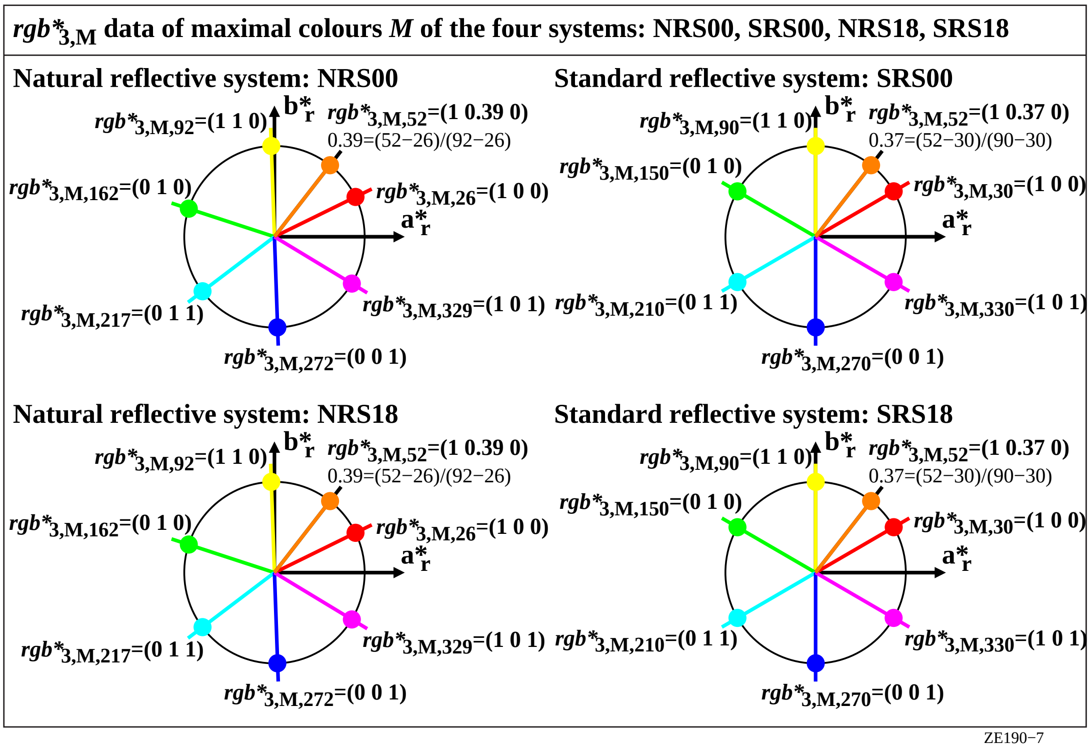

Figure 3: Relative *rgb$_3$ data of four systems NRS00, SRS00, NRS18 and SRS18**

Figure 3 shows the relative *rgb**$_3$ coordinates of the four systems NRS00, SRS00, NRS18 and SRS18. Both the systems NRS00 and NRS18 and SRS00 and SRS18 are equal in Fig. 3, which uses *relative* (r) coordinates (a^*_r, b^*_r) and *lab*rgb**$_3$.

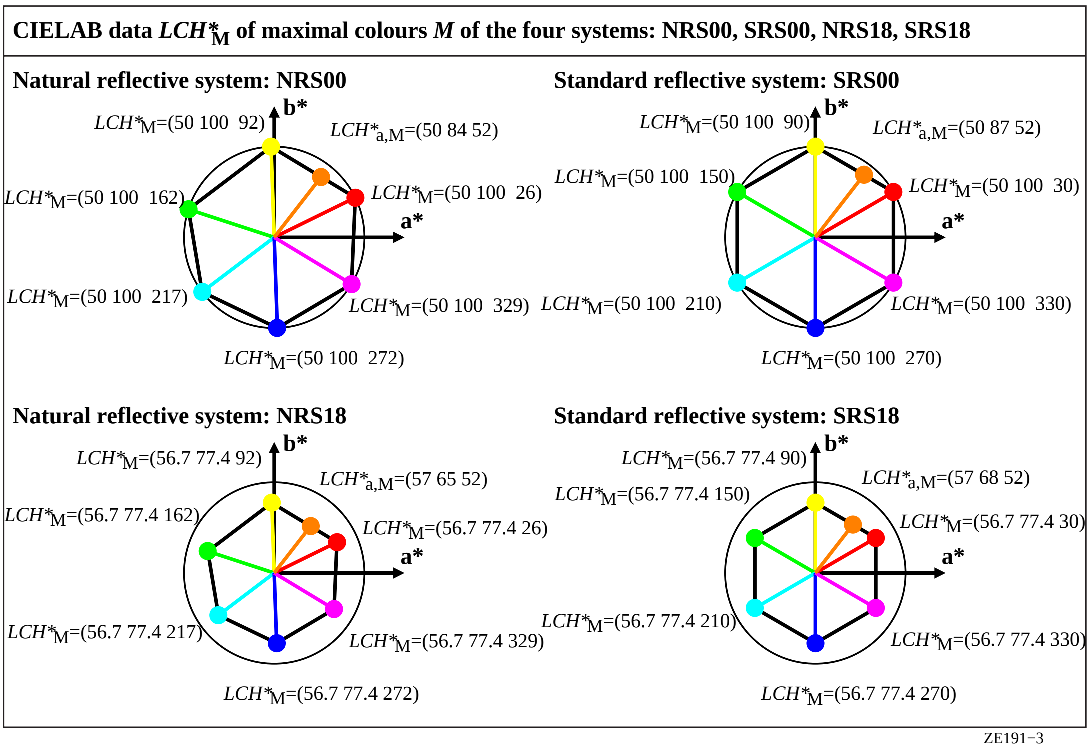

Figure 4: *LCH$_M$ data in CIELAB (a^*, b^*) diagram of four systems**

Figure 4 shows the LCH^*_M data of maximum colours *M* in the CIELAB (*a**, *b**) chroma diagram for the four systems NRS00, SRS00, NRS18 and SRS18. Both the systems NRS00 and SRS00 have the larger chroma of the value 100 compared to NRS18 and SRS18 with the chroma of the smaller value 77,4.

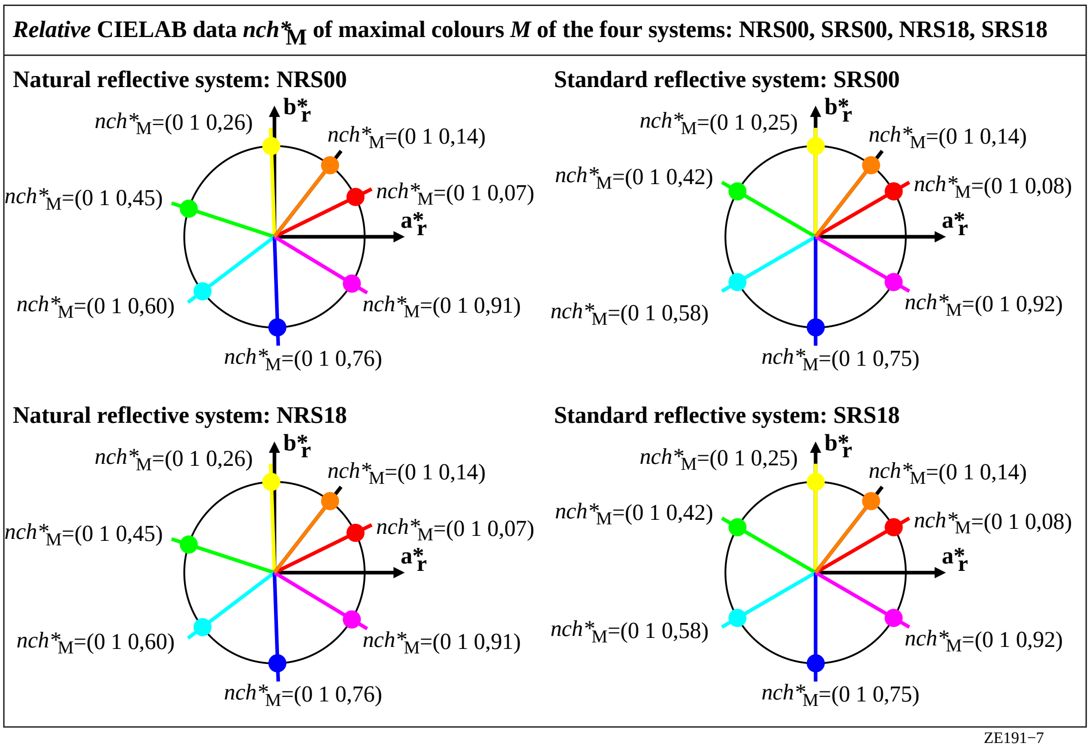

Figure 5: nch^*_M data of maximum colours *M* in the relative CIELAB (a^*_r, b^*_r) diagram

Figure 5 shows the nch^*_M data of maximum colours *M* in the relative CIELAB (a^*_r, b^*_r) diagram for the four systems NRS00, SRS00, NRS18 and SRS18. Both the systems NRS00 and NRS18 and SRS00 and SRS18 are equal in Fig. 5 which use *relative* (r) coordinates (a^*_r, b^*_r) and includes the *lab*nch** data.

3. Relative coordinates rgb^*_3 or *nce** and *linear* relation to CIELAB *LCH**

Colours can be specified by three coordinates for example in CIELAB by two sets of equivalent data *L*, a** and *b* (LAB*)* or *L*, C^*_{ab} and h_{ab} *(LCH*)*. In the following the set *L*, C^*_{ab} and h_{ab} is preferred and we call this set *LCH**.

Two new sets of coordinates rgb^*_3 and *nce** will be defined by linear relations to the CIELAB *LCH** data. A graphical relationship is shown first and later the equations are given in all directions of the following list. Therefore we will study the following three cases:

Data set given	**Data sets to be calculated**
*LCH**	*nce*, rgb^*_3*
rgb^*_3	*LCH*, nce**
*nce**	rgb^*_3, *LCH**

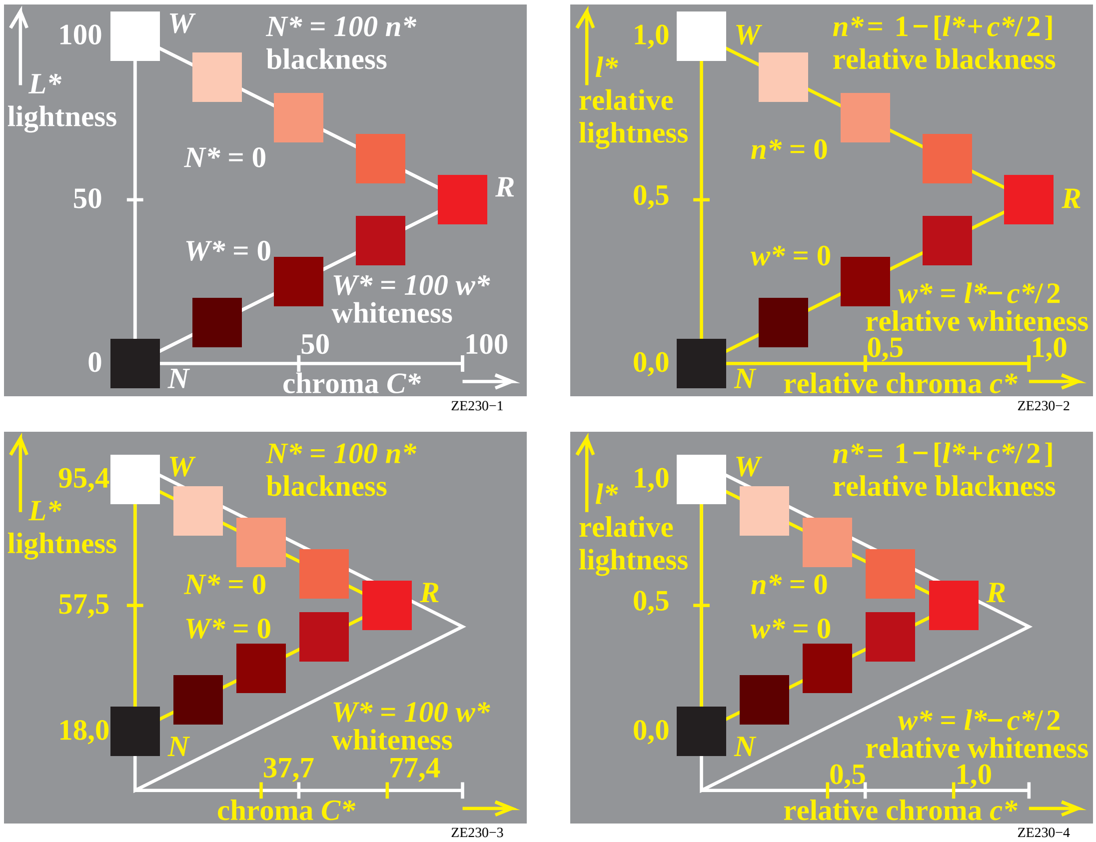

Figure 6: Colour of a red hue in CIELAB *(C^*_{ab}, L^*)* and relative *(c^*, l^*)* diagrams.
Figure 6 shows colours of a red hue in *absolute* CIELAB *(C^*_{ab}, L^*)* diagrams *(left)* and in *relative* CIELAB *(c^*, l^*)* diagrams *(right)*. The coordinates used are CIELAB chroma C^*_{ab}, CIELAB lightness L^*, blackness N^* and whiteness W^* (left) and *relative* CIELAB chroma c^*, *relative* CIELAB lightness l^*, *relative* blackness n^* and *relative* whiteness w^* *(right)*.

The following equations are valid with the lightness L^*_N and L^*_W of Black *N* and White *W*:

relative lightness	$l^* = [\, L^* - L^*_N \,] \,/\, [\, L^*_W - L^*_N \,]$	(1)
relative chroma	$c^* = C^*_{ab} \,/\, C^*_{ab,M}$	(2)
relative blackness	$n^* = 1 - [\, l^* + 0{,}5\, c^* \,]]$	(3)

Remark: If the lightness L^*_M of the maximum colour M is not equal to 0,5 ($L^*_W + L^*_N$) then the *relative* triangle lightness t^* instead of the *relative* lightness l^* must be used in this equation, see the equations in Fig. 12.

relative whiteness	$w^* = 1 - n^* - c^*$	(4)
blackness	$N^* = 100\, n^*$	(5)
whiteness	$W^* = 100\, w^*$	(6)

For the calculation of the intended relative coordinates rgb^*_3 and the elementary hue number e^* the CIELAB hue angle h_{ab} and its location relative to the elementary hues *RJGB* is necessary.

In a good approximation the four elementary hues are defined either by the six hue angles between 30 and 330 degrees or by the four CIE-test colours no. 9 to 12 of CIE 13.3 for *RJGB* and the intermediate hues C' = g50b and M' = b50r.

Elementary colour and CIE illuminant		CIELAB data, CIE tristimulus values and CIE chromaticity for the CIE standard illuminant D65 and D50 and the 2 degree observer									
CIE-test colour	Ill.	*L**	*a**	*b**	C^*_{ab}	h_{ab}	*X*	*Y*	*Z*	*x*	*y*
09, Red R	D65	40,04	58,98	28,32	65,43	25,7	20,64	11,27	4,34	0,5693	0,3110
10, Yellow J		81,30	−2,99	71,82	71,89	92,4	54,89	59,01	12,02	0,4359	0,4686
11, Green G		52,27	−42,40	13,64	44,54	162,2	12,15	20,38	15,34	0,2538	0,4258
12, Blue B		30,52	1,21	−46,35	46,37	271,5	6,24	6,45	27,59	0,1550	0,1601
09, Red R	D50	41,88	62,00	31,82	69,69	27,2	23,31	12,42	3,24	0,5982	0,3188
10, Yellow J		81,97	1,81	71,59	71,61	88,5	58,84	60,24	9,50	0,4576	0,4685
11, Green G		51,62	−41,12	11,52	42,70	164,4	12,10	19,81	11,95	0,2759	0,4515
12, Blue B		29,20	−5,28	−49,34	49,62	263,9	5,25	5,92	21,25	0,1621	0,1825

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Figure 7: CIELAB data of CIE-test colours for CIE standard illuminant D65 and D50
Figure 7 shows CIELAB data and other CIE data of the CIE-test colours no. 9 to 12 for the CIE standard illuminant D65 and the CIE illuminant D50.

The CIELAB hue angles of the elementary colours are h_{ab} = 25,7, 92,4, 162,2 and 271,5 degree. In some cases hue angles and CIELAB values which are intermediate between Green G and Blue B and intermediate between Blue B and Red R are necessary. These two colours are called Cyan blue dash C' = g50b an Magenta red dash M' = b50r.

In a first good approximation the hue angles of the six colours are located on a regular hexagon with 60 degree difference in the CIELAB *(a*, b*)* chroma diagram. For a better agreement with visual data instead of the hue angles between 30 and 330 the elementary hue angles 26, 92, 162, 217, 272 and 329 are used.

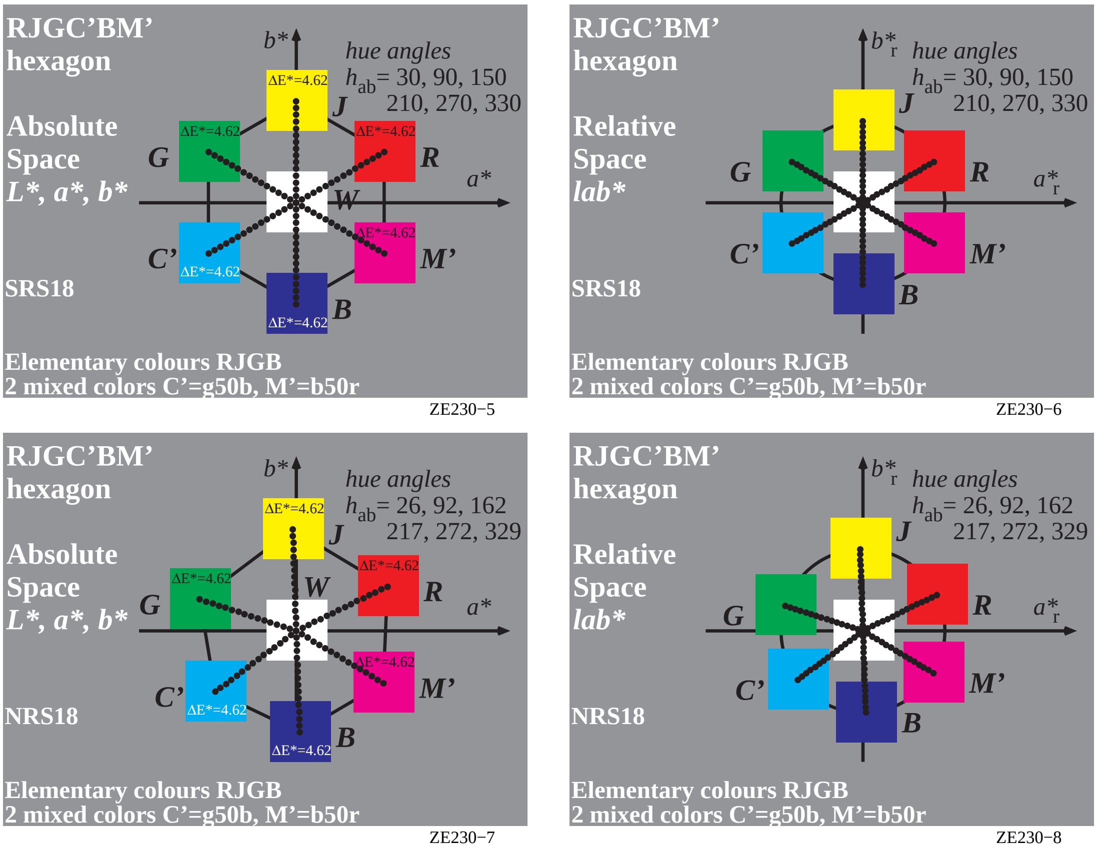

Figure 8: *Absolute* and *relative* CIELAB chroma data of systems SRS18 and NRS18

Figure 8 shows the *absolute* and *relative* CIELAB chroma data of the systems SRS18 and NRS18. The hue angles of the system SRS18 are regular between 30 and 330 degree. The system NRS18 uses the hue angles 26, 92, 162, 217, 272 and 329 degrees. Fig. 8 includes 16 step colour series between White W and the six chromatic colours. The CIELAB colour difference between two adjacent steps is calculated as $\Delta E^*_{ab} = 4{,}62$.

4. Interpretation of the rgb^*_3 coordinates and linear relation to *nicwd**

The elementary hues are used for the definition of the rgb^*_3 coordinates. The rgb^*_3 data have the three values 1 0 0, 0 1 0 and 0 0 1 for the maximum colours *X=R, G, B* of the elementary hues Red *R*, Green *G* and Blue *B*.

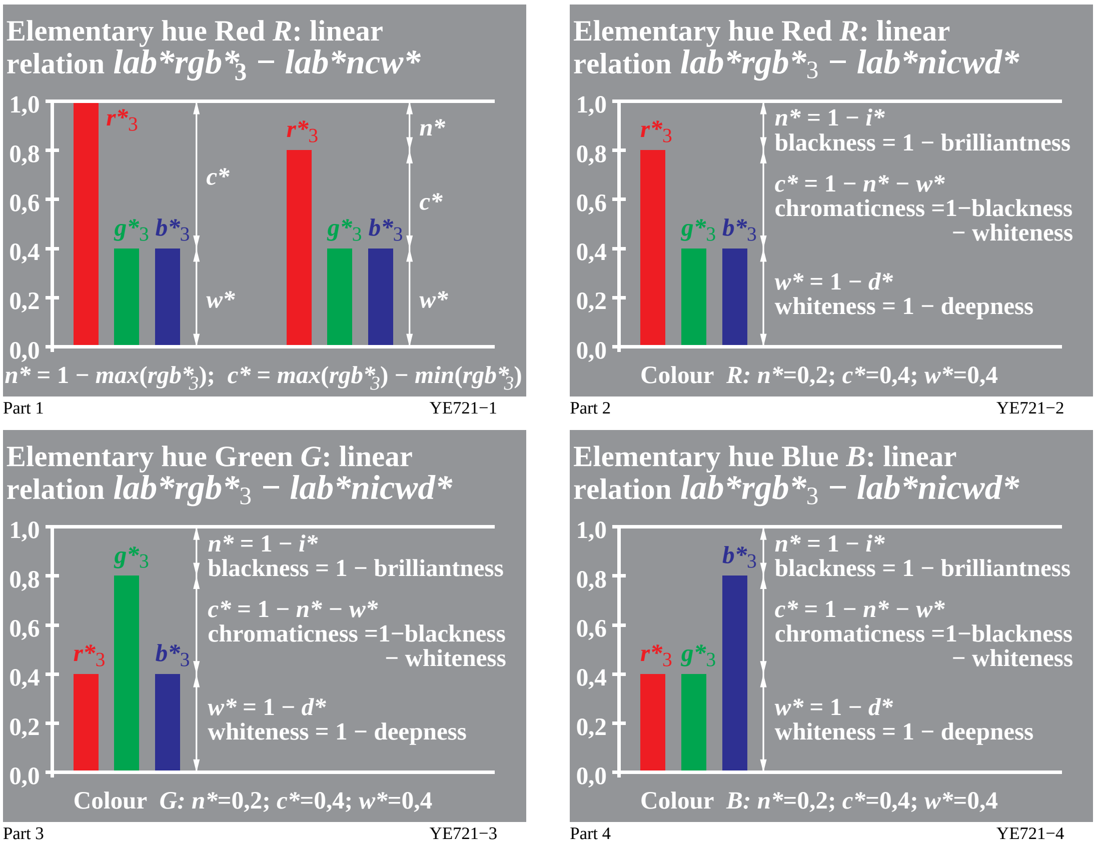

Figure 9: rgb^*_3 data and linear relation to *relative* blackness *n and *relative* chroma *c****

Figure 9 shows example rgb^*_3 data and the linear relation to *relative* blackness *n**, *relative* chroma *c**, *relative* whiteness *w*, relative* brilliantness *i** and *relative* colour deepness *d**.

Fig. 9 shows in part 1 and 2 colours of the elementary hue Red *R* and in part 3 and 4 colours of the elementary hues Green *G* and Blue *B*.

5. Examples of user friendly colour notation *nce** and linear relation to *rgb**$_3$

The colour notation *nce** is more user friendly compared to *rgb**$_3$ and we will study the linear relation between *nce** and *rgb**$_3$. Additionally the transformation between the elementary hue number *e** and the elementary (unique) hue text *u** and the CIELAB hue angle $h_{ab,M} = H^*_M$ is given.

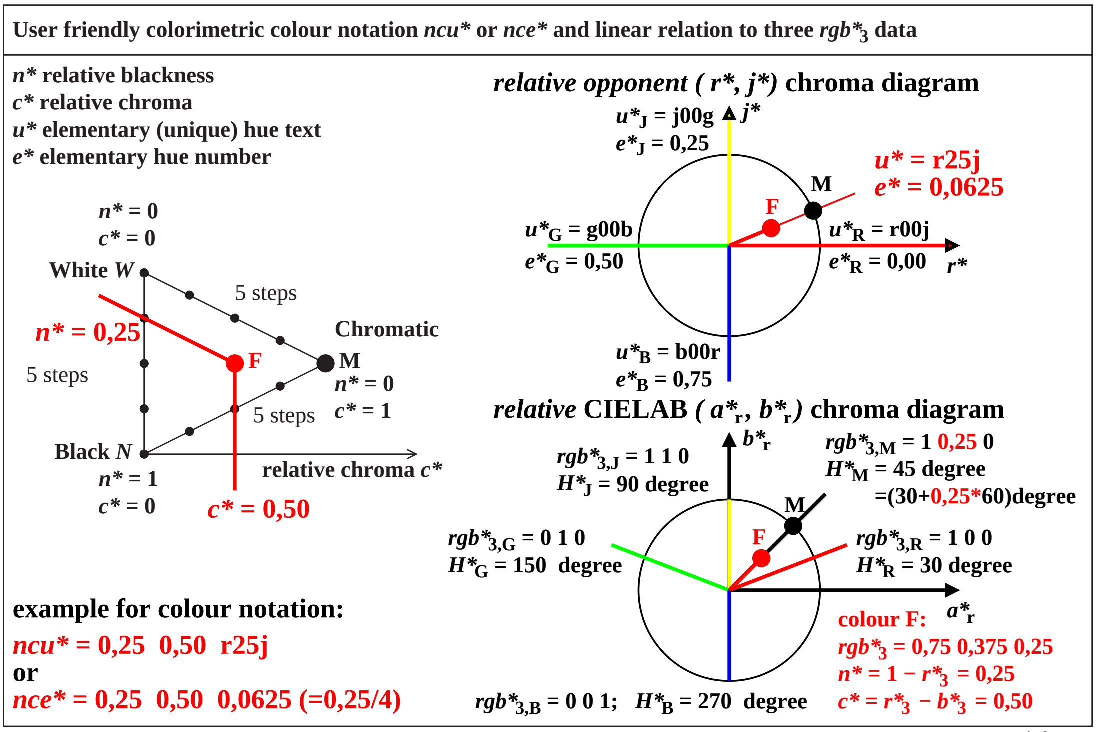

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Figure 10: Example *nce data and relation to *rgb**$_3$ data for the system SRS00**

Figure 10 shows example *nce** data and both the relation to *rgb**$_3$ data and to the CIELAB hue angle $H^*_M = h_{ab,M}$. The relation is equal for the systems SRS00 and SRS18. The *n** and *c** data define the largest and smallest value of the *rgb**$_3$ data. In the case that the maximum colour *M* is located between *R* and *J* it is valid for the example:

$$r^*_3 = 1 - n^* = 0{,}75$$

$$g^*_3 = 1 - n^* - c^* + c^*\, 0{,}01\, u^*_n$$

$$= w^* + c^*\, g^*_{3,M} = 0{,}25 + 0{,}50\; 0{,}25 = 0{,}375$$

$$b^*_3 = 1 - n^* - c^*$$

$$= w^* = 0{,}25$$

The number u^*_n changes between 00 and 99 for the hue series between elementary Red *R* (*u** = r00j) and nearly Yellow *J* (*u** =r99j).

The value of $g^*_{3,M}$ of the Maximum colour M is equal to the elementary hue text number u^*_n=25 divided by 100. Above and in Fig. 12 g^*_3 is calculated from *w**, *c**, and $g^*_{3,M}$.

The hue text number u^*_n is also used to calculate the CIELAB hue angle $h_{ab,M} = H^*_M$. In the example the CIELAB hue angle of M is located between the hue angles of *R* and *J*. Therefore it is valid

$$h_{ab,M} = h_{ab,R} + 0{,}01\, u^*_n\, [\, h_{ab,J} - h_{ab,R}\,] = 30 + 0{,}25\, [\, 90 - 30\,] = 45 = H^*_M$$

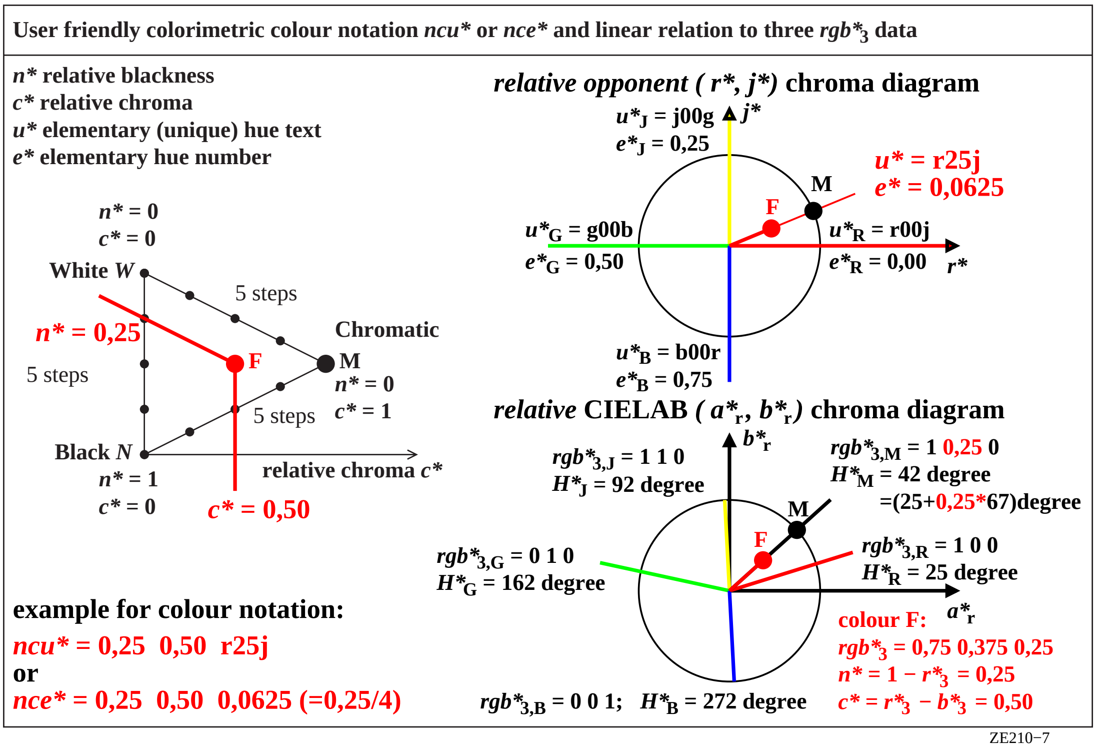

Figure 11: Example *nce data and relation to *rgb**$_3$ data for the system NRS00**

Figure 11 shows example *nce** data and both the relation to *rgb**$_3$ data and to the CIELAB hue angle $H^*_M = h_{ab,M}$. The relation is equal for the systems NRS00 and NRS18. Compared to Fig.10 the data *nce** data and the calculated *rgb**$_3$ data are the same but the hue angle $h_{ab,M}$ has changed. In the example the CIELAB hue angle of M is located between the new hue angles of *R* and *J*. Therefore it is valid

$$h_{ab,M} = h_{ab,R} + 0{,}01\ u^*_n\ [\ h_{ab,J} - h_{ab,R}\ ] = 25 + 0{,}25\ [\ 92 - 25\ ] = 42 = H^*_M$$

We conclude: According to the experimental results of *Evans* (1974) colours of cero *relative* blackness *n** and cero blackness *N** are located for all hues on a line which connects in the CIELAB space the white point (C^*_{ab}, L^*) = (0, 100) with the point (C^*_{ab}, L^*) = (100, 50).

In Fig. 10 and 11 an additional line connects the black point (C^*_{ab}, L^*) = (0, 0) with the point (C^*_{ab}, L^*) = (100, 50) and then a colour triangle in produced. The colour with the coordinate ($C^*_{ab,M}$, L^*_M) = (100, 50) is called the maximum colour *M*. The CIELAB chroma and the CIELAB lightness of the six colours *RJGC'BM* is equal, therefore:

CIELAB chroma of maximum colour *M*	$C^*_{ab,M} = 100$
CIELAB lightness of maximum colour *M*	$L^*_M = 50$

For any colour with any CIELAB chroma C^*_{ab} and any CIELAB lightness L^* of the same hue (h_{ab}=const) the equations (1) to (6) in section 3 are valid. These equations for example allow to calculate *relative* chroma *c**, *relative* lightness *l** and *relative* blackness *n**

6. Transformation between 3 sets of coordinates

We will transform between the three data sets *LCH**, *nce*, rgb**$_3$ in the following directions:

Data set given	Data sets to be calculated
*LCH**	*nce*, rgb**$_3$
*rgb**$_3$	*LCH**, *nce**
*nce**	*rgb**$_3$, *LCH**

Equations: colorimetric data transfer from *LCH (CIELAB) to *nce** and *rgb**$_3$**

Given: CIELAB data of any colour L^*, C^*_{ab}, h_{ab} = LCH^* = LAB^*LCH^* or L^*, a^*, b^*
CIELAB data L^*_X, $C^*_{ab,X}$, $h_{ab,X}$, a^*_X, b^*_X of eigth basic colours $X = RJGC'BM'NW$
Aim: *nce and *rgb**$_3$ elementary colour data of the given colour** (in example *M* located between *R* and *J*)

CIELAB Hue angle of maximum colour *M*	$h_{ab,M} = h_{ab}$ ($0 <= h_{ab} <= 360$)	(1)
Relative device hue angle ratio of *M*	$\alpha_M = [h_{ab,M} - h_{ab,R}] / [h_{ab,J} - h_{ab,R}]$	(2)
CIELAB data L^*_M, a^*_M, b^*_M, $C^*_{ab,M}$ of *M*	$L^*_M = \alpha_M L^*_J + (1 - \alpha_M) L^*_R$	(3)
	$a^*_M = \alpha_M a^*_J + (1 - \alpha_M) a^*_R$	(4)
	$b^*_M = \alpha_M b^*_J + (1 - \alpha_M) b^*_R$	(5)
	$C^*_{ab,M} = [a^*_M{}^2 + b^*_M{}^2]^{1/2}$	(6)
relative lightness of the given colour	$l^* = [L^* - L^*_N] / [L^*_W - L^*_N]$	(7)
relative chroma of the given colour	$c^* = C^*_{ab} / C^*_{ab,M}$	(8)
relative triangle lightness of the given colour	$t^* = l^* - [L^*_M - L^*_N] / [L^*_W - L^*_N] c^* + 0{,}5 c^*$	(9)
relative blackness of the given colour	$n^* = 1 - t^* - 0{,}5 c^*$	(10)
relative whiteness of the given colour	$w^* = 1 - n^* - c^*$	(11)
elementary hue angle of the given colour	e^* = function [h_{ab}] (with table or equation)	(12)
relative $rgb^*_{3,M}$ data of *M*	$r^*_{3,M} = \alpha_M r^*_{3,J} + (1 - \alpha_M) r^*_{3,R}$	(13)
	$g^*_{3,M} = \alpha_M g^*_{3,J} + (1 - \alpha_M) g^*_{3,R}$	(14)
	$b^*_{3,M} = \alpha_M b^*_{3,J} + (1 - \alpha_M) b^*_{3,R}$	(15)
relative rgb^*_3 data of the given colour	$r^*_3 = w^* + c^* r^*_{3,M}$	(16)
	$g^*_3 = w^* + c^* g^*_{3,M}$	(17)
	$b^*_3 = w^* + c^* b^*_{3,M}$	(18)

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Figure 12: Transfer from *LCH (CIELAB) data to *nce** and *rgb**$_3$ data**

Figure 12 shows the transformation between the *LCH** (CIELAB) data and the *relative* data *nce** and *rgb**$_3$. Figure 12 is based on the assumption, that the eight CIELAB data of the six chromatic and the two achromatic colours *X=RJGC'BM' NW* are given. The example assumes further that the given hue angle is between the hue angle of Red *R* and Yellow *J*. In other cases the colours *R* and *J* have to be replaced, for example by *J* and *G.*

Equations: colorimetric data transfer from *nce to elementary colour data rgb^*_3 and *LCH***		
Given: *nce data (similar NCS) of any colour *nce** = *lab*nce**** (in example *M* located between *R* and *J*)		
CIELAB data L^*_X, $C^*_{ab,X}$, $h_{ab,X}$, a^*_X, b^*_X of eigth basic colours *X = RJGC'BM'NW*		
Aim: Elementary colour data rgb^*_3 and *LCH (CIELAB) of the given colour**		
relative whiteness of the given colour	$w^* = 1 - n^* - c^*$	(1)
relative triangle lightness of the given colour	$t^* = \ = 1 - n^* - 0{,}5\ c^*$	(2)
CIELAB hue angle of maximum colour *M*	$h_{ab,M}$ = function [e^*] (with table or equation)	(3)
Relative device hue angle ratio of *M*	$\alpha_M = [h_{ab,M} - h_{ab,R}] / [h_{ab,J} - h_{ab,R}]$	(4)
relative $rgb^*_{3,M}$ data of *M*	$r^*_{3,M} = \alpha_M\ r^*_{3,J} + (1 - \alpha_M)\ r^*_{3,R}$	(5)
	$g^*_{3,M} = \alpha_M\ g^*_{3,J} + (1 - \alpha_M)\ g^*_{3,R}$	(6)
	$b^*_{3,M} = \alpha_M\ b^*_{3,J} + (1 - \alpha_M)\ b^*_{3,R}$	(7)
relative rgb^*_3 data of the given colour	$r^*_3 = w^* + c^*\ r^*_{3,M}$	(8)
	$g^*_3 = w^* + c^*\ g^*_{3,M}$	(9)
	$b^*_3 = w^* + c^*\ b^*_{3,M}$	(10)
CIELAB data L^*_M, a^*_M, b^*_M, $C^*_{ab,M}$ of *M*	$L^*_M = \alpha_M\ L^*_J + (1 - \alpha_M)\ L^*_R$	(11)
	$a^*_M = \alpha_M\ a^*_J + (1 - \alpha_M)\ a^*_R$	(12)
	$b^*_M = \alpha_M\ b^*_J + (1 - \alpha_M)\ b^*_R$	(13)
	$C^*_{ab,M} = [\ a^{*2}_M + b^{*2}_M\ ]^{1/2}$	(14)
relative lightness l^* of the given colour	$l^* = t^* + c^*\ [\ L^*_M - L^*_N\ ] / [\ L^*_W - L^*_N\ ] - 0{,}5\ c^*$	(15)
CIELAB data *LCH** of the given colour	$L^* = L^*_N + l^*\ [\ L^*_W - L^*_N\ ]$	(16)
	$C^*_{ab} = c^*\ C^*_{ab,M}$	(17)
	$h_{ab} = h_{ab,M}$	(18)

ZE211−3

Figure 13: Transfer from *nce* data* to rgb^*_3 data and *LCH (CIELAB) data**

Figure 13 shows the transformation from *nce** to rgb^*_3 data and CIELAB *LCH** data. The example assumes that the given hue angle defined by *e** is between the hue angle of Red *R* and Yellow *J*.

Equations: colorimetric data transfer from rgb^*_3 to *nce data and *LCH** data**		
Given: Elementary colour data of any colour rgb^*_3 = *lab*rgb* $_3$** (in example *M* located between *R* and *J*)		
CIELAB data L^*_X, $C^*_{ab,X}$, $h_{ab,X}$, a^*_X, b^*_X of eigth basic colours *X = RJGC'BM'NW*		
Aim: *nce = *lab*nce** (similar NCS) and *LCH** (CIELAB) data of the given colour** (0 <= e^* <= 1)		
relative chroma of the given colour	$c^* = max\ [\ rgb^*_3\ ] - min\ [\ rgb^*_3\ ]$	(1)
relative blackness of the given colour	$n^* = 1 - max\ [\ rgb^*_3\ ]$	(2)
relative triangle lightness of the given colour	$t^* = 1 - n^* - 0{,}5\ c^*$	(3)
relative red-green chroma in standard system s	$a^*_{rs} = r^*_3\ cos(30) + g^*_3\ cos(150)$	(4)
relative yellow-blue chroma in standard system s	$b^*_{rs} = r^*_3\ sin(30) + g^*_3\ sin(150) + b^*_3\ sin(270)$	(5)
hue angle in standard system s	$h_{ab,s} = arctan\ [\ b^*_{rs} / a^*_{rs}\ ]$ ($0 <= h_{ab,s} <= 360$)	(6)
CIELAB hue angle of maximum colour *M*	$h_{ab,M}$ = function [$h_{ab,s}$] (with table or equation)	(7)
elementary hue number of the given colour	e^* = function [$h_{ab,M}$] (with table or equation)	(8)
Relative device hue angle ratio of *M*	$\alpha_M = [h_{ab,M} - h_{ab,R}] / [h_{ab,J} - h_{ab,R}]$	(9)
CIELAB data L^*_M, a^*_M, b^*_M, $C^*_{ab,M}$ of *M*	$L^*_M = \alpha_M\ L^*_J + (1 - \alpha_M)\ L^*_R$	(10)
	$a^*_M = \alpha_M\ a^*_J + (1 - \alpha_M)\ a^*_R$	(11)
	$b^*_M = \alpha_M\ b^*_J + (1 - \alpha_M)\ b^*_R$	(12)
	$C^*_{ab,M} = [\ a^{*2}_M + b^{*2}_M\ ]^{1/2}$	(13)
relative lightness l^* of the given colour	$l^* = t^* + c^*\ [\ L^*_M - L^*_N\ ] / [\ L^*_W - L^*_N\ ] - 0{,}5\ c^*$	(14)
CIELAB data *LCH** of the given colour	$L^* = L^*_N + l^*\ [\ L^*_W - L^*_N\ ]$	(15)
	$C^*_{ab} = c^*\ C^*_{ab,M}$	(16)
	$h_{ab} = h_{ab,M}$	(17)

ZE211−7

Figure 14: Transfer from rgb^*_3 data to *nce and *LCH** (CIELAB) data**

Figure 14 shows the transformation from rgb^*_3 data to *nce** data and *LCH** (CIELAB) data.The example assumes that the given hue angle defined by rgb^*_3 is between the hue angle of Red *R* and Yellow *J*. In the case of a 60 degree system (either SRS00 or SRS18) the hue $h_{ab,M}$ of the maximum colour M is identical to $h_{ab,s}$ which is calculated by equations (4) to (6) in Fig. 14. In the case of the systems NRS00 and NRS18 the CIELAB hue angle $h_{ab,M}$ of the maximum colour M may be taken either from a table with 360 entries for $h_{ab,s}$ and the corresponding hue angle $h_{ab,M}$ or the CIELAB hue angle $h_{ab,M}$ may be calculated by equations, see Fig. 16.

Figures 12 to 14 are based on the assumption, that the eight CIELAB data of the six chromatic and the two achromatic colours *X=RJGC'BM' NW* are given. The example assumes further that the given hue angle is between the hue angle of Red *R* and Yellow *J*. In the general case the location of the given hue angle has to be compared with the six hue angles of *X=RJGC'BM'* and this defines the two neighbouring colours, for example *G* and *C'*, to be used for the calculations according to the equations in Fig. 12. to 14.

7. Transformation between CIELAB hue h_{ab} and elementary hue number *e**

The transformation between the CIELAB hue h_{ab} and the elementary hue number *e** or the elementary hue angle $h_{ab,e}$ will be studied.It is appropriate to use either equations or tables for the transformation from h_{ab} to *e** or the inverse direction.

Equations: colorimetric transfer from CIELAB hue angle h_{ab} to elementary hue number e^*			
Given: CIELAB hue angle h_{ab} ($0 <= h_{ab} <= 360$)			
CIELAB hue angles $h_{ab,eX}$ of four elementary colours $eX = RJGB$			
Aim: Elementary hue number e^* of the given colour ($0 <= e^* <= 1$)			
Calculate elementary hue angle $h_{ab,e}$ in one of five possible cases for h_{ab} ($0 <= h_{ab} <= 360$):			
If $0 <= h_{ab} < h_{ab,eR}$	$h_{ab,e} = 270 + 90\ [\ 360 + h_{ab} - h_{ab,eB}] / [360 + h_{ab,eR} - h_{ab,eB}]$		(1)
If $h_{ab,eR} <= h_{ab} < h_{ab,eJ}$	$h_{ab,e} = 0 + 90\ [h_{ab} - h_{ab,eR}] / [h_{ab,eJ} - h_{ab,eR}]$		(2)
If $h_{ab,eJ} <= h_{ab} < h_{ab,eG}$	$h_{ab,e} = 90 + 90\ [h_{ab} - h_{ab,eJ}] / [h_{ab,eG} - h_{ab,eJ}]$		(3)
If $h_{ab,eG} <= h_{ab} < h_{ab,eB}$	$h_{ab,e} = 180 + 90\ [h_{ab} - h_{ab,eG}] / [h_{ab,eB} - h_{ab,eG}]$		(4)
If $h_{ab,eB} <= h_{ab} < 360$	$h_{ab,e} = 270 + 90\ [h_{ab} - h_{ab,eB}] / [360 + h_{ab,eR} - h_{ab,eB}]$		(5)
Elementary hue number	$e^* = h_{ab,e} / 360$	($0 <= e^* <= 1$)	(6)
Inverse equations: transfer from elementary hue number e^* to CIELAB hue angle h_{ab}			
Given: elementary hue number e^* ($0 <= e^* <= 1$)			
CIELAB hue angles $h_{ab,eX}$ of four elementary colours $eX = RJGB$			
Aim: CIELAB hue angle h_{ab} of the given colour ($0 <= h_{ab} <= 360$)			
Elementary hue angle	$h_{ab,e} = 360\ e^*$	($0 <= e^* <= 1$)	(1i)
Calculate CIELAB hue angle h_{ab} in one of four possible cases for e^* ($0 <= e^* < 1$):			
If $0{,}00 <= e^* < 0{,}25$	$h_{ab} = h_{ab,eR} + [h_{ab,e} / 90\ ]\ [h_{ab,eJ} - h_{ab,eR}]$		(2i)
If $0{,}25 <= e^* < 0{,}50$	$h_{ab} = h_{ab,eJ} + [h_{ab,e} / 90 - 1{,}00]\ [h_{ab,eG} - h_{ab,eJ}]$		(3i)
If $0{,}50 <= e^* < 0{,}75$	$h_{ab} = h_{ab,eJ} + [h_{ab,e} / 90 - 2{,}00]\ [h_{ab,eG} - h_{ab,eJ}]$		(4i)
If $0{,}75 <= e^* < 1{,}00$	$h_{ab} = h_{ab,eJ} + [h_{ab,e} / 90 - 3{,}00]\ [h_{ab,eG} - h_{ab,eJ}]$		(5i)
only if $h_{ab} >= 360$ then:	$h_{ab} = h_{ab} - 360$	$0 <= h_{ab} <= 360$	(6i)

ZE200−3

Figure 15: Transformation between CIELAB hue h_{ab} and elementary hue *e**

Figure 15 shows the equations for the transformation between the CIELAB hue angle h_{ab} and the elementary hue number *e** and in the inverse direction. The solution depends on different cases for the CIELAB hue angle h_{ab} or the elementary hue number *e** in the inverse case, see Fig. 15.

8. Transformation between standard hue angle $h_{ab,s}$ and CIELAB hue angle h_{ab}

Inverse equations: transfer from standard hue angle $h_{ab,s}$ to CIELAB hue angle h_{ab}		
Given: standard hue angle $h_{ab,s}$ ($0 <= h_{ab,s} <= 360$)		
CIELAB hue angles $h_{ab,s,X}$ of six standard colours $sX = RJGC'BM'$		
Aim: CIELAB hue angle h_{ab} of the given colour ($0 <= h_{ab} <= 360$)		
Remark:		
The standard hue angle $h_{ab,s}$ is usually calculated from the data rgb^*_3		
relative red-green chroma in system s	$a^*_{rs} = r^*_3 \cos(30) + g^*_3 \cos(150)$	
relative yellow-blue chroma in system s	$b^*_{rs} = r^*_3 \sin(30) + g^*_3 \sin(150) + b^*_3 \sin(270)$	
hue angle in standard system s	$h_{ab,s} = \arctan [b^*_{rs} / a^*_{rs}]$	
Calculate CIELAB hue angle h_{ab} in one of seven possible cases for $h_{ab,s}$ ($0 <= h_{ab,s} < 360$):		
If $0 <= h_{ab,s} < 30$	$h_{ab} = h_{ab,s,M'} + [h_{ab,s}+360 - h_{ab,s,M'}] [h_{ab,s,R}+360 - h_{ab,s,M'}]/60$	(1i)
If $30 <= h_{ab,s} < 90$	$h_{ab} = h_{ab,s,R} + [h_{ab,s} - h_{ab,s,R}] [h_{ab,s,J} - h_{ab,s,R}]/60$	(2i)
If $90 <= h_{ab,s} < 150$	$h_{ab} = h_{ab,s,J} + [h_{ab,s} - h_{ab,s,J}] [h_{ab,s,G} - h_{ab,s,J}]/60$	(3i)
If $150 <= h_{ab,s} < 210$	$h_{ab} = h_{ab,s,G} + [h_{ab,s} - h_{ab,s,G}] [h_{ab,s,C'} - h_{ab,s,G}]/60$	(4i)
If $210 <= h_{ab,s} < 270$	$h_{ab} = h_{ab,s,C'} + [h_{ab,s} - h_{ab,s,C'}] [h_{ab,s,B} - h_{ab,s,C'}]/60$	(5i)
If $270 <= h_{ab,s} < 330$	$h_{ab} = h_{ab,s,B} + [h_{ab,s} - h_{ab,s,B}] [h_{ab,s,M'} - h_{ab,s,B}]/60$	(6i)
If $330 <= h_{ab,s} < 360$	$h_{ab} = h_{ab,s,M'}+360 + [h_{ab,s} - h_{ab,s,M'}] [h_{ab,s,R}+ 360 - h_{ab,s,M'}]/60$	(7i)
only if $h_{ab} >= 360$ then:	$h_{ab} = h_{ab} - 360$ $\quad 0 <= h_{ab} <= 360$	(8i)

ZE201−3

Figure 16: Transformation between standard hue angle $h_{ab,s}$ and CIELAB hue angle h_{ab}

Fig. 16 shows the transformation equations between the standard hue angle $h_{ab,s}$ and the CIELAB hue angle h_{ab}.

The equations of Fig. 16 are only necessary, if the *rgb**$_3$ data are given in the systems NRS00 or NRS18 with a non regular hue angle distribution. For NRS00 or NRS18 the hue angles are 26, 162 and 272 degrees for the elementary hues Red *R*, Green *G* and Blue *B*. In the regular case of the systems SRS00 or SRS18 the hue angles are 30, 150 and 270 degrees for the elementary hues Red *R*, Green *G* and Blue *B*. In this case the hue angle $h_{ab,s}$ is identical to the CIELAB hue angle h_{ab}. It is valid in this case:

$h_{ab,\,s} = h_{ab}$ (for SRS00 and SRS18)

Special tables include all three hue angles h_{ab}, $h_{ab,e}$ and $h_{ab,s}$ for the four systems (N/S)RS00 and (N/S)RS18 with a hue difference of $\Delta h_{ab} = 10$, see the URL (1 page, 50 kByte)

http://www.ps.bam.de/ZE22/10L/L22E00NP.PDF

According to *Miescher* (1969) the standard deviation for the determination of the elementary hues is four steps, if a 400 step hue circle of a mean high chroma C^*_{ab} about 80 is used. Therefore the use of three tables with the hue angle differences of $\Delta h_{ab} = 1$, $\Delta h_{ab,e} = 1$ and $\Delta h_{ab,e} = 1$ is appropriate for all applications.

For the hue tables for the systems SRS18 and SRS00 see the URL (4 pages,130 kByte)

http://www.ps.bam.de/ZE24/10L/L24E00NP.PDF

For the hue tables for the systems NRS18 and NRS00 see the URL (4 pages,130 kByte)

http://www.ps.bam.de/ZE25/10L/L25E00NP.PDF

9. Summary

Linear equations are shown which transform from *absolute* CIELAB *LCH** data to *relative* CIELAB *nce** and *rgb**$_3$ data if the CIELAB *LCH** data of eight reference colours are given. There are inverse transformations and therefore the coordinates *nce** and *rgb**$_3$ are device independent. The coordinates *nce** describe visual properties like *relative* blackness *n**, *relative* chroma *c**, and the elementary hue number *e**.

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