Weber-Fechner law in CIE 230:2019 for threshold colour differences of surface colours; relations between tristimulus value, luminance, and lightness

The *Weber-Fechner* law describes the lightness L^*_{rW} as ***logarithmic*** function of L_{rW}.
The *Stevens* law describes the lightness L^*_{CIELAB} as ***potential*** function of $L_{rW}=Y/90$.
$L^*_{CIELAB} = 116\ L_{rW}^{1/3} - 16 = 66\ L_{rU}^{1/3} - 16$, **Approximation:** $L^*_{IEC,sRGB} = 100\ L_{rW}^{1/2,4}$ [1]
The *Weber-Fechner* law is equivalent to the equation: $\Delta L_{rW} = c_W L_{rW}$ [2]
Integration leads to the logarithmic equation: $L^*_{rW} = t_W \log(L_{rW})$. [3]
Derivation leads for $\Delta L^*_{rW}=1$ to the linear equation: $L_{rW}/\Delta L_{rW} = t_W = 57$. [4]
For ***adjacent*** Colours in offices the standard contrast range is **25:1**=90:3,6.

Table 1: CIE tristimulus value *Y*, luminance *L*, and lightness *L

Colour (matte paper)	**tristimu-lus values**	**SDR office luminance**	**relative luminance**		**CIELAB$_W$ lightness**	**TUBLOG$_U$ lightness**
Contrast W:N (25:1=90:3,6)	***Y***	***L* [cd/m²]**	L_{rU} **=L/L_U**	L_{rW} **=L/L_W**	$L^*_{CIELAB\_W}$ **=$c_W L_{rW}^{1/3}-16$**	$L^*_{TUBLOG\_U}$ **=$t_U \log(L_{rU})+50$**
White W (paper)	90 =18*5	142 =28,2*5	5	1	96=50+46 =c(1)$^{1/3}$−16	100=50+50 =$t$log(5)+50
Grey U (paper)	18	28,2	1	0,2	49=50−1 =c(0,2)$^{1/3}$−16	50=50+0 =$t$log(1)+50
Black N (paper)	3,6 =18/5	5,6 28,2/5	0,2	0,04	22=50−28 =c(0,04)$^{1/3}$−16	0=50−50 =$t$log(0,2)+50
It is valid: CIELAB$_W$: c_W=c=116, TUBLOG$_U$: t_U=t=50/log(5)=57						

feo40−3n

Colourimetric scaling of achromatic colours between peak white and black. Relations between tristimulus value *Y*, luminance *L*, and lightness *L of ISO-standards**

Colour (light or paper)	**tristimu-lus values**	**HDR display luminance**	**relative luminance**		**CIELAB$_U$ lightness**	**TUBLOG$_U$ lightness**
Contrast W:N (25:1=90:3,6)	***Y*** ($5^{0,5}$=2,24	***L* [cd/m²]**	***L*** **=L/L_U**	L_{rW} **=L/L_W**	$L^*_{CIELAB_U}$ **=$d_U L_{rU}^{1/3}-16$**	$L^*_{TUBLOG_U}$ **=$t_U \log(L_{rU})+50$**
White P2 (light)	360 =18*20	800 =40*20	25	2,24	161=50+111 =d(11,2)$^{1/3}$−16	121=50+71 =t log(20)+50
White P1 (light)	180 =18*10	400 =40*10	20	1,00	125=50+75 =d(5,00)$^{1/3}$−16	104=50+54 =t log(10)+50
White W (fluo-recent paper)	90 =18*5	200 =40*5	5	0,45	95=50+45 =d(2,24)$^{1/3}$−16	87=50+37 =t log(5,0)+50
Grey U (paper)	18 =18*1	40 40*1	1	0,20	49=50−0 =d(1,00)$^{1/3}$−16	47=50−2 =t log(1)+50
Black N (paper)	3,6 =18/5	8 40/5	0,20	0,09	22=50−27 =d(0,45)$^{1/3}$−16	7=50−42 =t log(0,20)+50
Black p1 (glossy paper)	2,5 =18/7	5,7 40/7	0,14	0,04	17=50−32 =d(0,20)$^{1/3}$−16	−1=50−51 =t log(0,14)+50
Black p2 (glossy paper)	1,8 =18/10	4 40/10	0,10	0,022	14=50−35 =d(0,09)$^{1/3}$−16	−9=50−59 =t log(0,10)+50
It is valid: CIELAB$_U$: d_U=d=66, TUBLOG$_U$: t_U=t=40/log(5)=57						

feo41−3n

Colourimetric scaling of achromatic colours between peak white and black. Relations between tristimulus value *Y*, luminance *L*, and lightness *L of ISO-standards**

Colour (light or paper)	**tristimu-lus values**	**HDR display luminance**	**relative luminance**		**CIELAB$_W$ lightness**	**TUBLOG$_U$ lightness**
Contrast W:N (25:1=90:3,6)	***Y***	***L* [cd/m²]**	L_{rU} **=L/L_U**	L_{rW} **=L/L_W**	$L^*_{CIELAB_W}$ **=$c_W L_{rW}^{1/3}-16$**	$L^*_{TUBLOG_U}$ **=$t_U \log(L_{rU})+50$**
White P2 (light)	360 =18*20	800 =40*20	25	2,24	100=50+50 =c(2,00)$^{1/3}$−16	100=50+52 =t log(10,00)+50
White P1 (light)	180 =18*10	400 =40*10	20	1,00	76=50+26 =c(1,00)$^{1/3}$−16	78=50+30 =t log(5,00)+50
White W (fluo-recent paper)	90 =18*5	200 =40*5	5	0,45	54=50+4 =c(0,45)$^{1/3}$−16	53=50+5 =t log(2,24)+50
Grey U (paper)	18 =18*1	40 40*1	1	0,20	37=50−12 =c(0,20)$^{1/3}$−16	28=50−19 =t log(1,00)+50
Black N (paper)	3,6 =18/5	8 40/5	0,20	0,09	25=50−24 =c(0,09)$^{1/3}$−16	3=50−44 =t log(0,45)+50
Black p1 (glossy paper)	2,5 =18/7	5,7 40/7	0,14	0,04	15=50−34 =c(0,04)$^{1/3}$−16	−21=50−69 =t log(0,20)+50
Black p2 (glossy paper)	1,8 =18/10	4 40/10	0,10	0,022	8=50−41 =c(0,02)$^{1/3}$−16	−43=50−91 =t log(0,10)+50
It is valid: CIELAB$_W$: c_W=c=116, TUBLOG$_U$: t_U=t=50/log(5)=72						

feo40−7n

Colourimetric scaling of achromatic colours between peak white and black. Relations between tristimulus value *Y*, luminance *L*, and lightness *L of ISO-standards**

Colour (light or paper)	**tristimu-lus values**	**HDR display luminance**	**relative luminance**		**IECsRGB$_W$ lightness**	**TUBLOG$_U$ lightness**
Contrast W:N (25:1=90:3,6)	***Y*** ($5^{0,5}$=2,24	***L* [cd/m²]**	***L*** **=L/L_U**	L_{rW} **=L/L_W**	$L^*_{IECsRGB_W}$ **=$s_W L_{rW}^{1/2,4}$**	$L^*_{TUBLOG_U}$ **=$t_U \log(L_{rU})+50$**
White P2 (light)	360 =18*20	800 =40*20	25	2,24	170=50+120 =s(2,24)$^{1/2,4}$	121=50+71 =t log(20)+50
White P1 (light)	180 =18*10	400 =40*10	20	1,00	127=50+77 =s(1,00)$^{1/2,4}$	104=50+54 =t log(10)+50
White W (fluo-recent paper)	90 =18*5	200 =40*5	5	0,45	95=50+45 =s(0,45)$^{1/2,4}$	87=50+37 =t log(5,0)+50
Grey U (paper)	18 =18*1	40 40*1	1	0,20	48=50−1 =s(0,20)$^{1/2,4}$	47=50−2 =t log(1)+50
Black N (paper)	3,6 =18/5	8 40/5	0,20	0,09	25=50−24 =s(0,09)$^{1/2,4}$	7=50−42 =t log(0,20)+50
Black p1 (glossy paper)	2,5 =18/7	5,7 40/7	0,14	0,04	21=50−28 =s(0,04)$^{1/2,4}$	−1=50−51 =t log(0,14)+50
Black p2 (glossy paper)	1,8 =18/10	4 40/10	0,10	0,022	18=50−31 =s(0,02)$^{1/2,4}$	−9=50−59 =t log(0,10)+50
It is valid: IECsRGB$_W$: s_W=s=100, TUBLOG$_U$: t_U=t=40/log(5)=57						

feo41−7n