Weber-Fechner law in CIE 230:2019 for threshold colour differences of surface colours; relations between tristimulus value, luminance, and lightness

The *Weber-Fechner* law describes the lightness L^*_{rW} as ***logarithmic*** function of L_{rW}.
The *Stevens* law describes the lightness L^*_{CIELAB} as ***potential*** function of $L_{rW}=Y/90$.
$L^*_{CIELAB} = 116\ L_{rW}^{1/3} - 16 = 66\ L_{rU}^{1/3} - 16$, **Approximation:** $L^*_{IEC,sRGB} = 100\ L_{rW}^{1/2,4}$ [1]
The *Weber-Fechner* law is equivalent to the equation: $\Delta L_{rW} = c_W L_{rW}$ [2]
Integration leads to the logarithmic equation: $L^*_{rW} = t_W \log(L_{rW})$. [3]
Derivation leads for $\Delta L^*_{rW}=1$ to the linear equation: $L_{rW}/\Delta L_{rW} = t_W = 57$. [4]
For ***adjacent*** Colours in offices the standard contrast range is **25:1**=90:3,6.

Table 1: CIE tristimulus value *Y*, luminance *L*, and lightness *L

Colour (matte paper)	tristimu-lus values	SDR office luminance	relative luminance		$CIELAB_W$ lightness	$TUBLOG_U$ lightness
Contrast W:N (25:1=90:3,6)	***Y***	***L* [cd/m²]**	$L_{rU}=L/L_U$	$L_{rW}=L/L_W$	$L^*_{CIELAB\_W}=c_W L_{rW}^{1/3}-16$	$L^*_{TUBLOG\_U}=t_U \log(L_{rU})+50$
White W (paper)	90 =18*5	142 =28,2*5	5	1	96=50+46 =c(1)$^{1/3}$−16	90=50+40 =$t$log(5)+50
Grey U (paper)	18	28,2	1	0,2	49=50−1 =c(0,2)$^{1/3}$−16	50=50+0 =$t$log(1)+50
Black N (paper)	3,6 =18/5	5,6 28,2/5	0,2	0,04	22=50−28 =c(0,04)$^{1/3}$−16	10=50−40 =$t$log(0,2)+50
It is valid: $CIELAB_W$: c_W=c=116, $TUBLOG_U$: t_U=t=50/log(5)=72						

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Colourimetric scaling of achromatic colours between peak white and black. Relations between tristimulus value *Y*, luminance *L*, and lightness *L of ISO-standards**

Colour (light or paper)	tristimu-lus values	HDR display luminance	relative luminance		$CIELAB_W$ lightness	$TUBLOG_U$ lightness
Contrast W:N (25:1=90:3,6)	***Y***	***L* [cd/m²]**	$L_{rU}=L/L_U$	$L_{rW}=L/L_W$	$L^*_{CIELAB_W}=c_W L_{nW}^{1/3}-16$	$L^*_{TUBLOG_U}=t_U \log(L_{nU})+52$
White P2 (light)	360 =18*20	800 =40*20	25	2,24	161=50+111 =c(4,00)$^{1/3}$−16	141=50+91 =t log(20,00)+52
White P1 (light)	180 =18*10	400 =40*10	20	1,00	125=50+75 =c(2,00)$^{1/3}$−16	120=50+70 =t log(10,00)+52
White W (fluo-recent paper)	90 =18*5	200 =40*5	5	0,45	95=50+45 =c(1,00)$^{1/3}$−16	98=50+48 =t log(5,00)+52
Grey U (paper)	18 =18*1	40 40*1	1	0,20	49=50−0 =c(0,20)$^{1/3}$−16	48=50−1 =t log(1,00)+52
Black N (paper)	3,6 =18/5	8 40/5	0,20	0,09	22=50−27 =c(0,04)$^{1/3}$−16	−1=50−51 =t log(0,20)+52
Black p1 (glossy paper)	2,5 =18/7	5,7 40/7	0,14	0,04	17=50−32 =c(0,03)$^{1/3}$−16	−12=50−62 =t log(0,14)+52
Black p2 (glossy paper)	1,8 =18/10	4 40/10	0,10	0,022	14=50−35 =c(0,02)$^{1/3}$−16	−22=50−72 =t log(0,10)+52
It is valid: $CIELAB_W$: c_W=c=116, $TUBLOG_U$: t_U=t=50/log(5)=72						

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Colourimetric scaling of achromatic colours between peak white and black. Relations between tristimulus value *Y*, luminance *L*, and lightness *L of ISO-standards**

Colour (light or paper)	tristimu-lus values	HDR display luminance	relative luminance		$CIELAB_U$ lightness	$TUBLOG_U$ lightness
Contrast W:N (25:1=90:3,6)	***Y***	***L* [cd/m²]**	$L_{rU}=L/L_U$	$L_{rW}=L/L_W$	$L^*_{CIELAB_U}=d_U L_{nU}^{1/3}-16$	$L^*_{TUBLOG_U}=t_U \log(L_{nU})+52$
White P2 (light)	360 =18*20	800 =40*20	25	2,24	161=50+111 =c(20,00)$^{1/3}$−16	141=50+91 =t log(20,00)+52
White P1 (light)	180 =18*10	400 =40*10	20	1,00	125=50+75 =c(10,00)$^{1/3}$−16	120=50+70 =t log(10,00)+52
White W (fluo-recent paper)	90 =18*5	200 =40*5	5	0,45	95=50+45 =c(5,00)$^{1/3}$−16	98=50+48 =t log(5,00)+52
Grey U (paper)	18 =18*1	40 40*1	1	0,20	49=50−0 =c(1,00)$^{1/3}$−16	48=50−1 =t log(1,00)+52
Black N (paper)	3,6 =18/5	8 40/5	0,20	0,09	22=50−27 =c(0,20)$^{1/3}$−16	−1=50−51 =t log(0,20)+52
Black p1 (glossy paper)	2,5 =18/7	5,7 40/7	0,14	0,04	17=50−32 =c(0,14)$^{1/3}$−16	−12=50−62 =t log(0,14)+52
Black p2 (glossy paper)	1,8 =18/10	4 40/10	0,10	0,022	14=50−35 =c(0,10)$^{1/3}$−16	−22=50−72 =t log(0,10)+52
It is valid: $CIELAB_U$: d_U=d=66, $TUBLOG_U$: t_U=t=50/log(5)=72						

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Colourimetric scaling of achromatic colours between peak white and black. Relations between tristimulus value *Y*, luminance *L*, and lightness *L of ISO-standards**

Colour (light or paper)	tristimu-lus values	HDR display luminance	relative luminance		$IECsRGB_W$ lightness	$TUBLOG_U$ lightness
Contrast W:N (25:1=90:3,6)	***Y***	***L* [cd/m²]**	$L_{rU}=L/L_U$	$L_{rW}=L/L_W$	$L^*_{IECsRGB_W}=s_W L_{nW}^{1/2,4}$	$L^*_{TUBLOG_U}=t_U \log(L_{nU})+52$
White P2 (light)	360 =18*20	800 =40*20	25	2,24	170=50+120 =s(4,00)$^{1/2,4}$	141=50+91 =t log(20,00)+52
White P1 (light)	180 =18*10	400 =40*10	20	1,00	127=50+77 =s(2,00)$^{1/2,4}$	120=50+70 =t log(10,00)+52
White W (fluo-recent paper)	90 =18*5	200 =40*5	5	0,45	95=50+45 =s(1,00)$^{1/2,4}$	98=50+48 =t log(5,00)+52
Grey U (paper)	18 =18*1	40 40*1	1	0,20	48=50−1 =s(0,20)$^{1/2,4}$	48=50−1 =t log(1,00)+52
Black N (paper)	3,6 =18/5	8 40/5	0,20	0,09	25=50−24 =s(0,04)$^{1/2,4}$	−1=50−51 =t log(0,20)+52
Black p1 (glossy paper)	2,5 =18/7	5,7 40/7	0,14	0,04	21=50−28 =s(0,03)$^{1/2,4}$	−12=50−62 =t log(0,14)+52
Black p2 (glossy paper)	1,8 =18/10	4 40/10	0,10	0,022	18=50−31 =s(0,02)$^{1/2,4}$	−22=50−72 =t log(0,10)+52
It is valid: $IECsRGB_W$: s_W=s=100, $TUBLOG_U$: t_U=t=50/log(5)=72						

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