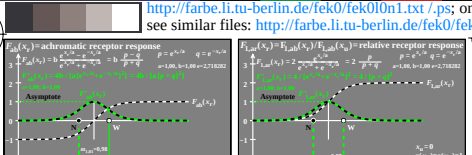


Mathematical equations of hyperbel functions
See: Papula, L., (2003), Mathematische Formelsammlung, Vieweg

$$F_{ab}(x/a) = b\,\tanh(x/a) = b\,\frac{e^{x/a}-e^{-x/a}}{e^{x/a}+e^{-x/a}} = b\,\frac{u(x)}{v(x)} \quad [1]$$

$$F'_{ab}(x/a) = b\,\frac{u'(x)\,v(x) - u(x)\,v'(x)}{v^2(x)} \quad [2]$$

$$F'_{ab}(x/a) = b\,\frac{v^2(x) - u^2(x)}{a\,v^2(x)} \quad [3]$$

$$F'_{ab}(x/a) = \frac{4b}{a\,[e^{x/a}+e^{-x/a}]^2} = \frac{b}{a\,\cosh^2(x/a)} \quad [4]$$

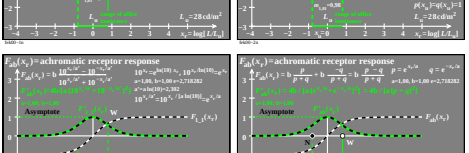
Mathematical equations of hyperbel functions
See: Papula, L., (2003), Mathematische Formelsammlung, Vieweg

$$F_{abu}(x/a) = [\tanh(x/a)+1] \,/\, [\tanh(x_n/a)+1] \quad [1u]$$

$$F'_{abu}(x/a) = \tanh(x/a) \text{ with } \tanh(x_n/a)=0 \quad [2u]$$

$$F'_{abu}(x/a) = \frac{v^2(x) - u^2(x)}{a\,v^2(x)} \quad [3u]$$

$$F'_{abu}(x/a) = \frac{4}{a\,[e^{x/a}+e^{-x/a}]^2} = \frac{1}{a\,\cosh^2(x/a)} \quad [4u]$$



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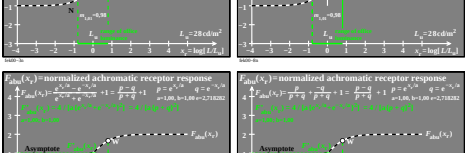

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$$F_{ab}(x/a) = b\,\tanh(x/a) = b\,\frac{e^{x/a}-e^{-x/a}}{e^{x/a}+e^{-x/a}} \quad [1]$$

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$$\frac{dF_{ab}(x_r/a)}{dx_r} = \frac{4b}{a\,[e^{x_r/a}+e^{-x_r/a}]^2} \qquad x_r = \log(L/L_n) \quad [5]$$

$$dx_r/dL = \ln(10)/L$$

$$\frac{dF_{ab}(x_r/a)}{dx_r} \cdot \frac{dx_r}{dL} = \frac{4b}{a\,[e^{x_r/a}+e^{-x_r/a}]^2} \cdot \frac{\ln(10)}{L} \quad [6]$$

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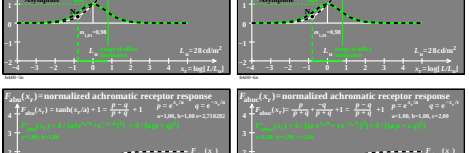

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Mathematical equations of hyperbel functions
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TUB-test chart fek0; Model of normalized response function $F_{ab}(x_r)$ and derivation $F'_{ab}(x_r)$
Mathematical calculation of the derivation $F'_{ab}(x_r)$, of the contrast $L/\Delta L$, and the discrimination ΔL

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