Line-element examples for grey samples ($0{,}2 \le x \le 5$)

F(x) is called the line-element function of ***f(x)***.
The following relations are valid for $x=Y/Y_u=Y/18$:

$$\frac{d[F(x)]}{dx} = f(x) \quad [1]$$

$$F(x) = \int \frac{f'(x)}{f(x)}\,dx \quad [2]$$

Example for the normalized tristimulus value $x=Y/Y_u$:

$$\frac{d[\mathrm{a}\ln(1+\mathrm{b}x)]}{dx} = \frac{\mathrm{ab}}{1+\mathrm{b}x} \quad [3]$$

$$\mathrm{a}\ln(1+\mathrm{b}x) = \int \frac{\mathrm{ab}}{1+\mathrm{b}x}\,dx \quad [4]$$

eeo00−1n DEQ60−1N

Line-element examples for grey samples ($0{,}2 \le x \le 5$)

$F_u(x)$ is called the line-element function of ***$f_u(x)$***.
Both functions are normalized to the surround value:

$$\frac{d[F_u(x)]}{dx} = f_u(x) \quad [1]$$

$$F_u(x) = \int \frac{f'_u(x)}{f_u(x)}\,dx \quad [2]$$

Example for the normalized functions with $x_u=1$:

$$F_u(x) = \frac{F(x)}{F(x_u)} = \frac{\ln(1+\mathrm{b}x)}{\ln(1+\mathrm{b})} \quad [3]$$

$$f_u(x) = \frac{f(x)}{f(x_u)} = \frac{1+\mathrm{b}x}{1+\mathrm{b}} \quad [4]$$

eeo00−2n DEQ60−2N

Line-element examples for grey samples ($0{,}2 \le x \le 5$)

$F_u(x)$ is called the line-element function of ***$f_u(x)$***.
Both functions are normalized to the surround value:

$$\frac{d[F_u(x)]}{dx} = f_u(x) \quad [1]$$

$$F_u(x) = \int \frac{f'_u(x)}{f_u(x)}\,dx = \int \frac{\mathrm{b}}{1+\mathrm{b}x}\,dx \quad [2]$$

Example for $L^*(x)$ & ΔY with $x=Y/Y_u$, $x_u=1$, b=6,141:

$$L^*_u(x) = \frac{L^*(x)}{L^*(x_u)} = \frac{\ln(1+\mathrm{b}x)}{\ln(1+\mathrm{b})} \quad [3]$$

$$f_u(x) = \frac{\Delta Y}{\Delta Y_u} = \frac{1+\mathrm{b}x}{1+\mathrm{b}} \quad [4]$$

eeo00−3n DEQ60−3N

Line-element equations according to CIE 230:2019

Colour-threshold (t) function $f_t(x) = \Delta Y_t = \Delta x\, Y_u$ [0]
$\Delta Y_t=(A_1+A_2Y)/A_0$ A_0=1,5, A_1=0,0170, A_2=0,0058

$$f_{tu}(x) = \frac{\Delta Y_t}{\Delta Y_{tu}} = \frac{1+\mathrm{b}x}{1+\mathrm{b}} \quad \mathrm{b}=A_2Y_u/A_1 \quad x=Y/Y_u \quad [1]$$

$$F_{tu}(x) = \int \frac{f'_{tu}(x)}{f_{tu}(x)}\,dx = \int \frac{\mathrm{b}}{1+\mathrm{b}x}\,dx \quad [2]$$

Example for $L^*_{tu}(x)$, ΔY_t with $x=Y/Y_u$, $x_u=1$, b=6,141:

$$L^*_{tu}(x) = \frac{L^*_t(x)}{L^*_{tu}(x)} = \frac{\ln(1+\mathrm{b}x)}{\ln(1+\mathrm{b})} \quad [3]$$

$$f_{tu}(x) = \frac{\Delta Y_t}{\Delta Y_{tu}} = \frac{1+\mathrm{b}x}{1+\mathrm{b}} \quad [4]$$

eeo00−4n DEQ60−4N

Line-element equations according to CIE 230:2019

Colour-discrimination function $f(x) = \Delta Y = \Delta x\, Y_u$ [0]
$\Delta Y=(A_1+A_2Y)/A_0$ A_0=1,5, A_1=0,0170, A_2=0,0058

$$f_u(x) = \frac{\Delta Y}{\Delta Y_u} = \frac{1+\mathrm{b}x}{1+\mathrm{b}} \quad \mathrm{b}=A_2Y_u/A_1 \quad x=Y/Y_u \quad [1]$$

$$F_u(x) = \int \frac{f'_u(x)}{f_u(x)}\,dx = \int \frac{\mathrm{b}}{1+\mathrm{b}x}\,dx \quad [2]$$

Example for $L^*(x)$ & ΔY with $x=Y/Y_u$, $x_u=1$, b=6,141:

$$L^*_u(x) = \frac{L^*(x)}{L^*(x_u)} = \frac{\ln(1+\mathrm{b}x)}{\ln(1+\mathrm{b})} \quad [3]$$

$$f_u(x) = \frac{\Delta Y}{\Delta Y_u} = \frac{1+\mathrm{b}x}{1+\mathrm{b}} \quad [4]$$

eeo00−5n DEQ60−5N

Line-element equations for thresholds and scaling

Colour-discrimination function $f(x) = \Delta Y = \Delta x\, Y_u$ [0]
$\Delta Y=1/[(1+x)(2+x)]=1/[1+x]-1/[2+x]$ $x=\sqrt{2}\,e^{k(u-u_0)}$

$$f_u(x) = \frac{\Delta Y}{\Delta Y_u} = \frac{1+\mathrm{b}x}{1+\mathrm{b}} - \frac{1+0{,}5\mathrm{b}x}{1+0{,}5\mathrm{b}} \quad \mathrm{b}=1,\ x=Y/Y_u \quad [1]$$

$$F_u(x) = \int \frac{f'_u(x)}{f_u(x)}\,dx = \int \frac{\mathrm{b}}{1+\mathrm{b}x}\,dx - \int \frac{0{,}5\mathrm{b}}{1+0{,}5\mathrm{b}x}\,dx \quad [2]$$

Example for $L^*(x)$ & ΔY with $x=Y/Y_u$, x_u=1, b=1:

$$L^*_u(x) = \frac{L^*(x)}{L^*(x_u)} = \frac{\ln(1+\mathrm{b}x)}{\ln(1+\mathrm{b})} - \frac{\ln(1+0{,}5\mathrm{b}x)}{\ln(1+0{,}5\mathrm{b})} \quad [3]$$

$$f_u(x) = \frac{\Delta Y}{\Delta Y_u} = \frac{1+\mathrm{b}x}{1+\mathrm{b}} - \frac{1+0{,}5\mathrm{b}x}{1+0{,}5\mathrm{b}} \quad [4]$$

see K. Richter (1996), Computer Graphic and Colorimetry, p. 113−127
http://color.li.tu-berlin.de/BUA4BF.PDF

eeo00−6n DEQ60−6N

Line-element equations for thresholds and scaling

Colour-discrimination function $f(x) = \Delta Y = \Delta x\, Y_u$ [0]
$\Delta Y=1/[(1+x)(2+x)]=1/[1+x]-1/[2+x]$ $x=\sqrt{2}\,e^{k(u-u_0)}$

$$f_u(x) = \frac{\Delta Y}{\Delta Y_u} = \frac{1+x}{2} - \frac{2+x}{3} \quad x=Y/Y_u \quad [1]$$

$$F_u(x) = \int \frac{f'_u(x)}{f_u(x)}\,dx = \int \frac{1}{1+x}\,dx - \int \frac{1}{2+x}\,dx \quad [2]$$

Example for $L^*(x)$ & ΔY with $x=Y/Y_u$, x_u=1:

$$L^*_u(x) = \frac{L^*(x)}{L^*(x_u)} = \frac{\ln(1+x)}{\ln(2)} - \frac{\ln(1+0{,}5x)}{\ln(1{,}5)} \quad [3]$$

$$f_u(x) = \frac{\Delta Y}{\Delta Y_u} = \frac{1+x}{2} - \frac{1+0{,}5x}{1{,}5} \quad [4]$$

see K. Richter (1996), Computer Graphic and Colorimetry, p. 113−127
http://color.li.tu-berlin.de/BUA4BF.PDF

eeo00−7n DEQ60−7N

Line-element equations for thresholds and scaling

Colour-discrimination function $f(y) = \Delta Y = \Delta y\, Y_u$ [0]
$\Delta Y=1/[y(1+y)]=1/y-1/(1+y)$ $y=(1+\sqrt{2}\,e^{k(u-u_0)}$

$$f_u(y) = \frac{\Delta Y}{\Delta Y_u} = \frac{y}{2} - \frac{1+y}{3} \quad y=1+Y/Y_u,\ dy=dx \quad [1]$$

$$F_u(y) = \int \frac{f'_u(y)}{f_u(y)}\,dy = \int \frac{1}{y}\,dy - \int \frac{1}{1+y}\,dy \quad [2]$$

Example for $L^*(y)$ & ΔY with $y=1+Y/Y_u$, y_u=2:

$$L^*_u(y) = \frac{L^*(y)}{L^*(y_u)} = \frac{\ln(y)}{\ln(2)} - \frac{\ln(1+y)}{\ln(3)} \quad [3]$$

$$f_u(y) = \frac{\Delta Y}{\Delta Y_u} = \frac{1+x}{2} - \frac{1+0{,}5x}{1{,}5} \quad [4]$$

see K. Richter (1996), Computer Graphic and Colorimetry, p. 113−127
http://color.li.tu-berlin.de/BUA4BF.PDF

eeo00−8n DEQ60−8N

Line-element examples for grey samples ($0{,}2 \le Y_r \le 5$)

$F(Y_r)$ is called the line-element function of ***$f(Y_r)$***.
The following relations are valid for $Y_r=Y/Y_u=Y/18$:

$$\frac{d[F(Y_r)]}{dY_r} = f(Y_r) \quad [1]$$

$$F(Y_r) = \int \frac{f'(Y_r)}{f(Y_r)}\,dY_r \quad [2]$$

Example for the normalized tristimulus value $Y_r=Y/Y_u$:

$$\frac{d[\mathrm{a}\ln(1+\mathrm{b}Y_r)]}{dY_r} = \frac{\mathrm{ab}}{1+\mathrm{b}Y_r} \quad [3]$$

$$\mathrm{a}\ln(1+\mathrm{b}Y_r) = \int \frac{\mathrm{ab}}{1+\mathrm{b}Y_r}\,dY_r \quad [4]$$

eeo01−1n DEQ61−1N

Line-element examples for grey samples ($0{,}2 \le Y_r \le 5$)

$F_u(Y_r)$ is called the line-element function of ***$f_u(Y_r)$***.
Both functions are normalized to the surround value:

$$\frac{d[F_u(Y_r)]}{dY_r} = f_u(Y_r) \quad [1]$$

$$F_u(Y_r) = \int \frac{f'_u(Y_r)}{f_u(Y_r)}\,dY_r \quad [2]$$

Example for the normalized functions with Y_r=1:

$$F_u(Y_r) = \frac{F(Y_r)}{F(1)} = \frac{\ln(1+\mathrm{b}Y_r)}{\ln(1+\mathrm{b})} \quad [3]$$

$$f_u(Y_r) = \frac{f(Y_r)}{f(1)} = \frac{1+\mathrm{b}Y_r}{1+\mathrm{b}} \quad [4]$$

eeo01−2n DEQ61−2N

Line-element examples for grey samples ($0{,}2 \le Y_r \le 5$)

$F_u(Y_r)$ is called the line-element function of ***$f_u(Y_r)$***.
Both functions are normalized to the surround value:

$$\frac{d[F_u(Y_r)]}{dY_r} = f_u(Y_r) \quad [1]$$

$$F_u(Y_r) = \int \frac{f'_u(Y_r)}{f_u(Y_r)}\,dY_r = \int \frac{\mathrm{b}}{1+\mathrm{b}Y_r}\,dY_r \quad [2]$$

Example for $L^*(Y_r)$ & ΔY_r with Y_u=1, b=6,141:

$$L^*_u(Y_r) = \frac{L^*(Y_r)}{L^*(Y_{ru})} = \frac{\ln(1+\mathrm{b}Y_r)}{\ln(1+\mathrm{b})} \quad [3]$$

$$f_u(Y_r) = \frac{\Delta Y_r}{\Delta Y_{ru}} = \frac{1+\mathrm{b}Y_r}{1+\mathrm{b}} \quad [4]$$

eeo01−3n DEQ61−3N

Line-element equations according to CIE 230:2019

Colour-threshold (t) function $f_t(Y_r) = \Delta Y_t = \Delta Y_r Y_u$ [0]
$\Delta Y_t=(A_1+A_2Y)/A_0$ A_0=1,5, A_1=0,0170, A_2=0,0058

$$f_{tu}(Y_r) = \frac{\Delta Y_t}{\Delta Y_{tu}} = \frac{1+\mathrm{b}Y_r}{1+\mathrm{b}} \quad \mathrm{b}=A_2Y_u/A_1 \quad Y_r=Y/Y_u \quad [1]$$

$$F_{tu}(Y_r) = \int \frac{f'_{tu}(Y_r)}{f_{tu}(Y_r)}\,dY_r = \int \frac{\mathrm{b}}{1+\mathrm{b}Y_r}\,dY_r \quad [2]$$

Example for $L^*_{tu}(Y_r)$, ΔY_t with $Y_{ru}=Y_r/Y_u$=1, b=6,141:

$$L^*_{tu}(Y_r) = \frac{L^*_t(Y_r)}{L^*_t(Y_{ru})} = \frac{\ln(1+\mathrm{b}Y_r)}{\ln(1+\mathrm{b})} \quad [3]$$

$$f_{tu}(Y_r) = \frac{\Delta Y_t}{\Delta Y_{tu}} = \frac{1+\mathrm{b}Y_r}{1+\mathrm{b}} \quad [4]$$

eeo01−4n DEQ61−4N

Line-element equations according to CIE 230:2019

Colour-discrimination function $f(Y_r) = \Delta Y_r$ [0]
$\Delta Y_r=(A_1+A_2Y_r)/A_0$ A_0=1,5, A_1=0,0170, A_2=0,0058

$$f_u(Y_r) = \frac{\Delta Y_r}{\Delta Y_{ru}} = \frac{1+\mathrm{b}Y_r}{1+\mathrm{b}} \quad \mathrm{b}=A_2Y_u/A_1 \quad Y_r=Y/Y_u \quad [1]$$

$$F_u(Y_r) = \int \frac{f'_u(Y_r)}{f_u(Y_r)}\,dY_r = \int \frac{\mathrm{b}}{1+\mathrm{b}Y_r}\,dY_r \quad [2]$$

Example for $L^*(Y_r)$ & ΔY_r with $Y_{ru}=Y_r/Y_u$=1, b=6,141:

$$L^*_u(Y_r) = \frac{L^*(Y_r)}{L^*(Y_{ru})} = \frac{\ln(1+\mathrm{b}Y_r)}{\ln(1+\mathrm{b})} \quad [3]$$

$$f_u(Y_r) = \frac{\Delta Y_r}{\Delta Y_{ru}} = \frac{1+\mathrm{b}Y_r}{1+\mathrm{b}} \quad [4]$$

eeo01−5n DEQ61−5N

Line-element equations for thresholds and scaling

Colour-discrimination function $f(Y_r)=\Delta Y_r$, $u_r=\ln Y_r$ [0]
$\Delta Y_r=1/[(1+Y_r)(2+Y_r)]=1/[1+Y_r]-1/[2+Y_r]$ $Y_r=\sqrt{2}\,e^{ku_r}$

$$f_u(Y_r) = \frac{\Delta Y_u}{\Delta Y_{ru}} = \frac{1+\mathrm{b}Y_r}{1+\mathrm{b}} - \frac{1+0{,}5\mathrm{b}Y_r}{1+0{,}5\mathrm{b}} \quad \mathrm{b}=1,\ Y_r=Y/Y_u \quad [1]$$

$$F_u(Y_r) = \int \frac{f'_u(Y_r)}{f_u(Y_r)}\,dY_r = \int \frac{\mathrm{b}\,dY_r}{1+\mathrm{b}Y_r} - \int \frac{0{,}5\mathrm{b}\,dY_r}{1+0{,}5\mathrm{b}Y_r} \quad [2]$$

Example for $L^*(Y_r)$ & ΔY with $Y_r=Y/Y_u$, Y_{ru}=1, b=1:

$$L^*_u(Y_r) = \frac{L^*(Y_u)}{L^*(Y_{ru})} = \frac{\ln(1+\mathrm{b}Y_r)}{\ln(1+\mathrm{b})} - \frac{\ln(1+0{,}5\mathrm{b}Y_r)}{\ln(1+0{,}5\mathrm{b})} \quad [3]$$

$$f_u(Y_r) = \frac{\Delta Y_r}{\Delta Y_{ru}} = \frac{1+\mathrm{b}Y_r}{1+\mathrm{b}} - \frac{1+0{,}5\mathrm{b}Y_r}{1+0{,}5\mathrm{b}} \quad [4]$$

see K. Richter (1996), Computer Graphic and Colorimetry, p. 113−127
http://color.li.tu-berlin.de/BUA4BF.PDF

eeo01−6n DEQ61−6N

Line-element equations for thresholds and scaling

Colour-discrimination function $f(Y_r)=\Delta Y_r$, $u_r=\ln Y_r$ [0]
$\Delta Y_r=1/[(1+Y_r)(2+Y_r)]=1/[1+Y_r]-1/[2+Y_r]$ $Y_r=\sqrt{2}\,e^{ku_r}$

$$f_u(Y_u) = \frac{\Delta Y_r}{\Delta Y_{ru}} = \frac{1+Y_r}{2} - \frac{2+Y_r}{3} \quad Y_r=Y/Y_u \quad [1]$$

$$F_u(Y_r) = \int \frac{f'_u(Y_r)}{f_u(Y_r)}\,dY_r = \int \frac{dY_r}{1+Y_r} - \int \frac{dY_r}{2+Y_r} \quad [2]$$

Example for $L^*(Y_r)$ & ΔY_r with $Y_r=Y/Y_u$=1:

$$L^*_u(Y_r) = \frac{L^*(Y_r)}{L^*(Y_{ru})} = \frac{\ln(1+Y_r)}{\ln(2)} - \frac{\ln(1+0{,}5Y_r)}{\ln(1{,}5)} \quad [3]$$

$$f_u(Y_r) = \frac{\Delta Y_r}{\Delta Y_{ru}} = \frac{1+Y_r}{2} - \frac{1+0{,}5Y_r}{1{,}5} \quad [4]$$

see K. Richter (1996), Computer Graphic and Colorimetry, p. 113−127
http://color.li.tu-berlin.de/BUA4BF.PDF

eeo01−7n DEQ61−7N

Line-element equations for thresholds and scaling

Colour-discrimination function $f(y) = \Delta Y = \Delta y\, Y_u$ [0]
$\Delta Y=1/[y(1+y)]=1/y-1/(1+y)$ $y=1+\sqrt{2}\,e^{k(u_r)}$, $u_r=\ln Y_r$

$$f_u(y) = \frac{\Delta Y}{\Delta Y_u} = \frac{y}{2} - \frac{1+y}{3} \quad y=1+Y/Y_u,\ dy=dx \quad [1]$$

$$F_u(y) = \int \frac{f'_u(y)}{f_u(y)}\,dy = \int \frac{1}{y}\,dy - \int \frac{1}{1+y}\,dy \quad [2]$$

Example for $L^*(y)$ & ΔY with $y=1+Y/Y_u$, y_u=2:

$$L^*_u(y) = \frac{L^*(y)}{L^*(y_u)} = \frac{\ln(y)}{\ln(2)} - \frac{\ln(1+y)}{\ln(3)} \quad [3]$$

$$f_u(y) = \frac{\Delta Y}{\Delta Y_u} = \frac{1+Y_r}{2} - \frac{1+0{,}5Y_r}{1{,}5} \quad [4]$$

see K. Richter (1996), Computer Graphic and Colorimetry, p. 113−127
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TUB registration: 20230701-eeo0/eeo0l0n1.txt /.ps
application for evaluation and measurement of display or print output
TUB material: code=rha4ta

TUB-test chart eeo0; CIE Y and lightness L^* for surface colours and for light-display colours
Line-element optimization of the colour difference formula LABJND according to CIE 230:2019