

# **Deductive and inductive antagonistic TUB colorimetry to improve CIE colorimetry for wide ranges of luminance and chromatic adaptation**

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## **Introduction**

In surface and screen color applications, the CIELAB color space and the associated CIELAB color-space formula are usually used. To describe the color differences, CIE 230:2019 recommends the color difference formula CIEDE2000\_PF for all color differences and LABJND\_PF for small color differences . Both lack an associated color space.

The *deductive* and *inductive* antagonistic TUB colorimetry contains possibilities to improve and extend the CIE colorimetry for a wide range of luminance and chromatic adaptation.

TUB colorimetry uses *physiological* data in the retina of monkeys and psychophysical data on luminance thresholds as well as hue thresholds of *Ostwald* optimal colors.

The TUB model provides a *two-step* colorimetry, which calculates two colorimetric coordinates  $L^*$  and  $F^*$  by mathematically integrating the luminance threshold  $dL$  and the contrast ( $L/dL$ ). The mathematical derivative of  $L^*$  and  $F^*$  again calculates  $dL$  and  $(L/dL)$ .

## Physiological receptor response of monkeys as a function of test-field luminance at three adaptation luminances

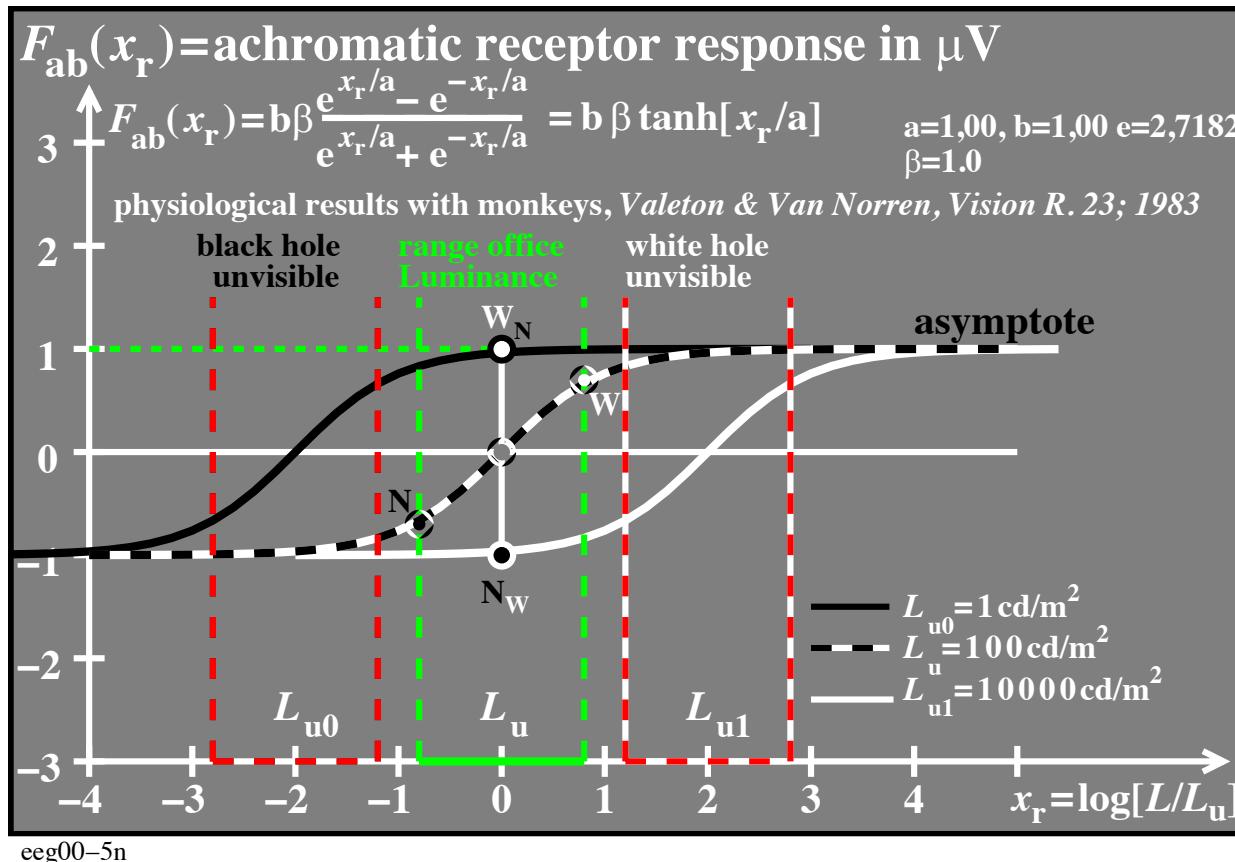
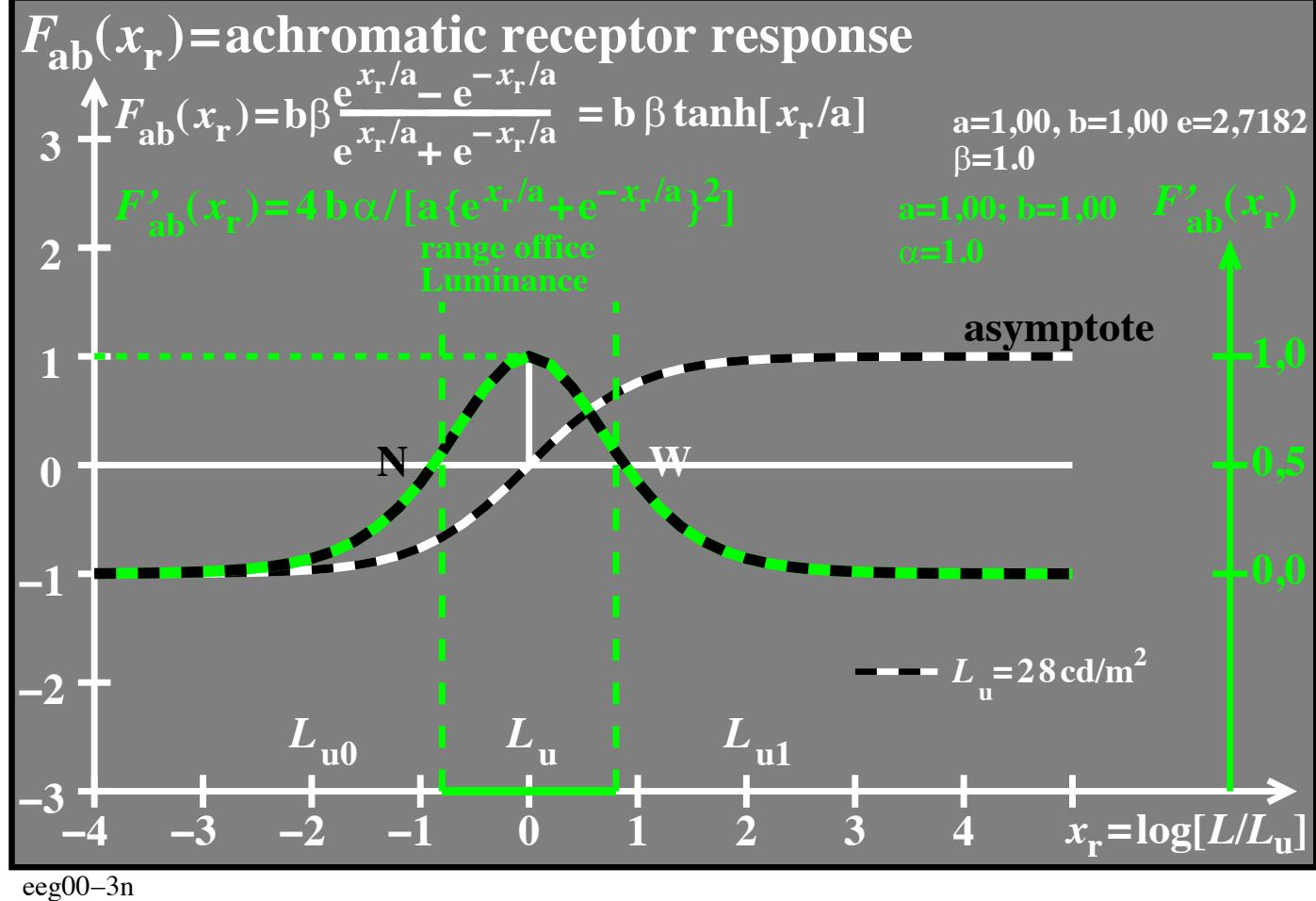


Figure 1: Physiological receptor signals for 3 surround-luminance adaptations  $L_u$ .



**Figure 2: Physiological receptor signal and mathematical derivation describing the signal or response difference**

## Mathematical equations of hyperbel functions

See: Papula, L., (2003), *Mathematische Formelsammlung*, Vieweg

$$F_{ab}(x_r/a) = b \tanh(x_r/a) = b \frac{e^{x_r/a} - e^{-x_r/a}}{e^{x_r/a} + e^{-x_r/a}} \quad [1]$$

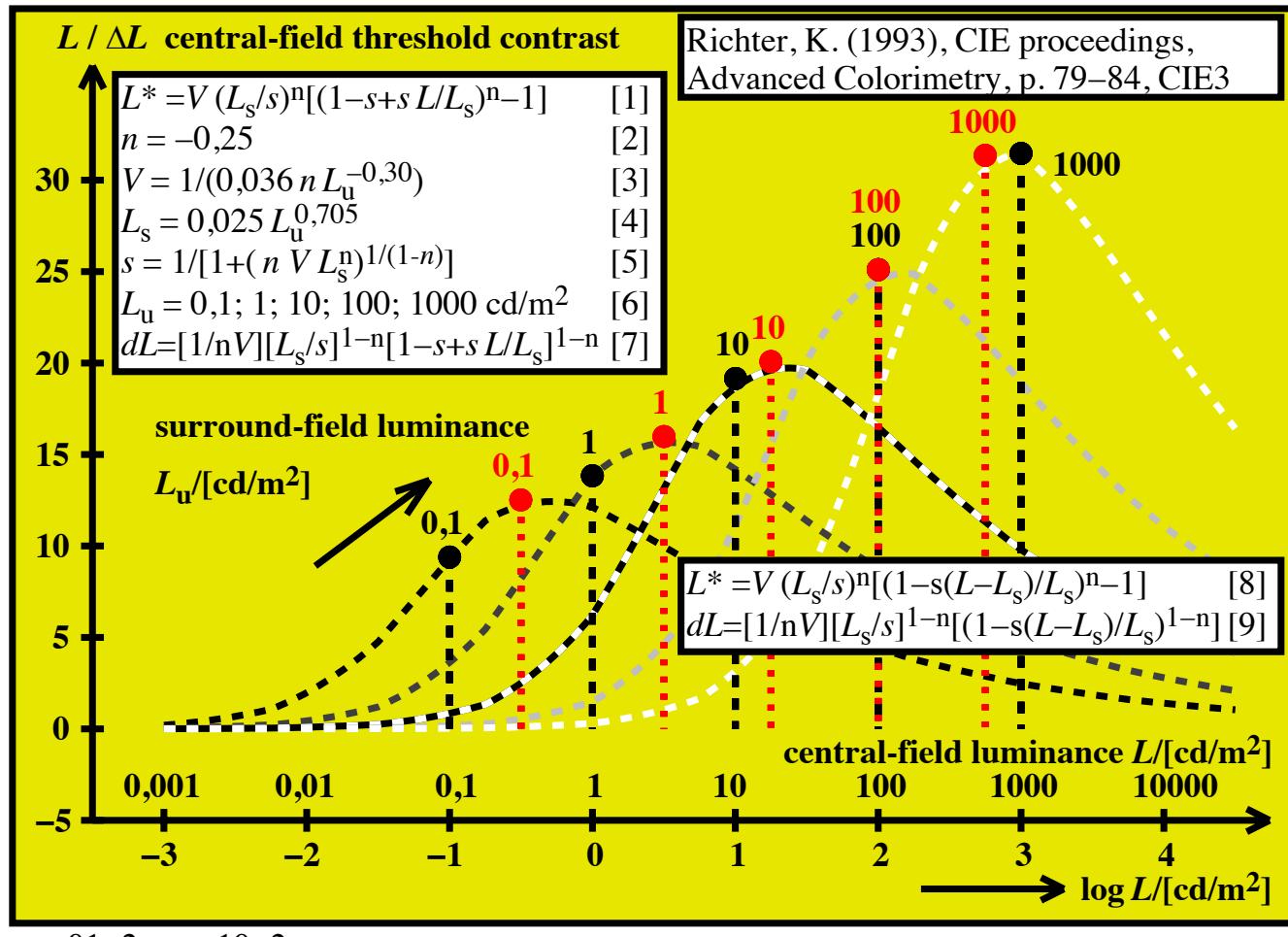
$$\frac{dF_{ab}(x_r/a)}{dx_r} = \frac{4b}{a[e^{x_r/a} + e^{-x_r/a}]^2} \quad x_r = \log(L/L_u) \quad [5]$$
$$dx_r/dL = \ln(10)/L$$

$$\frac{dF_{ab}(x_r/a)}{dx_r} \frac{dx_r}{dL} = \frac{4b}{a[e^{x_r/a} + e^{-x_r/a}]^2} \frac{\ln(10)}{L} \quad [6]$$

$$\frac{L}{dL} = \frac{4b\ln(10)}{a[e^{x_r/a} + e^{-x_r/a}]^2} \quad dL = \frac{a[e^{x_r/a} + e^{-x_r/a}]^2 L}{4b\ln(10)} \quad [7]$$

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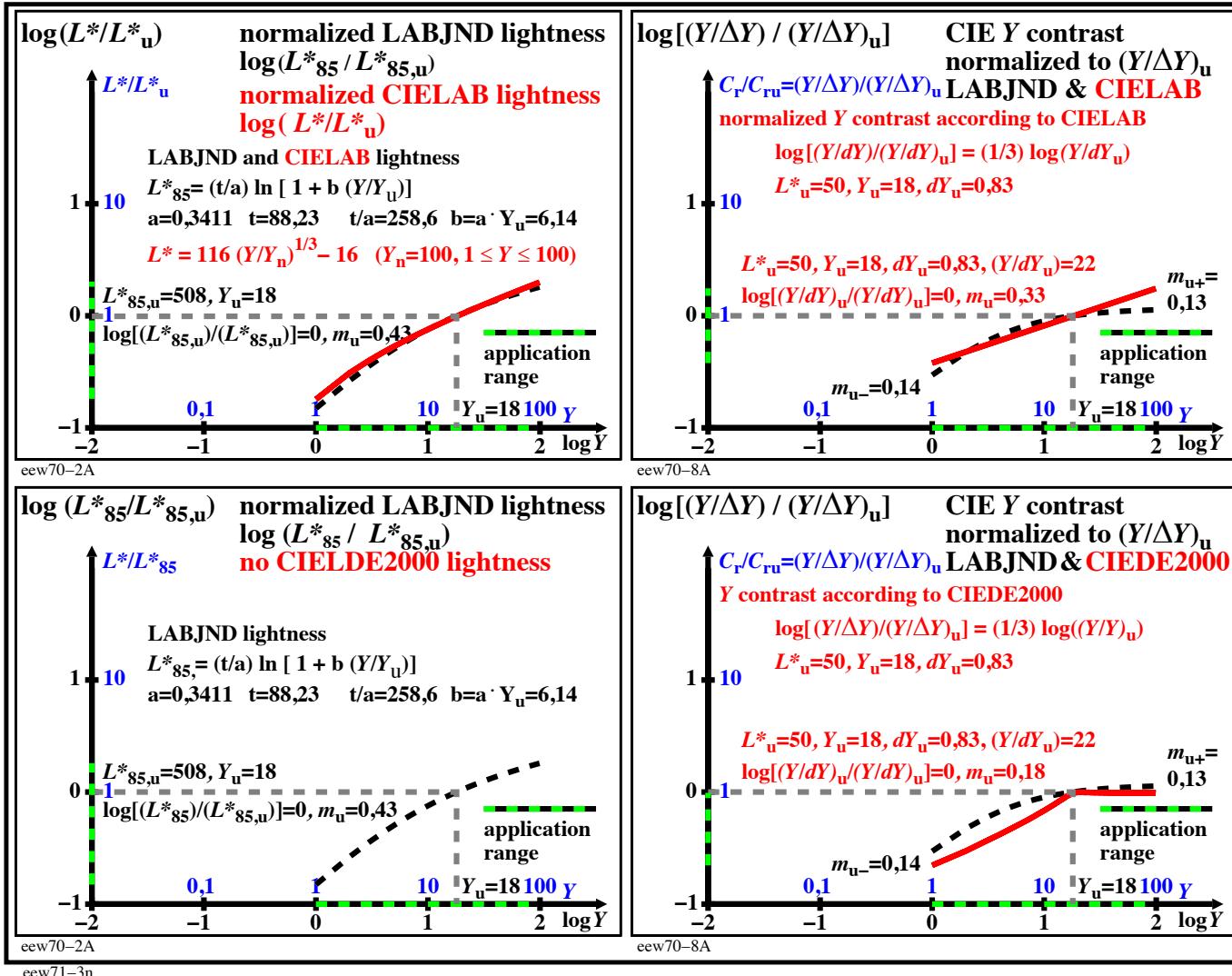
Figure 3: Receptor signal  $F$ , and equality of the derivative  $F'$  and the contrast  $L/dL$



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**Figure 4 Contrast thresholds  $L/dL$  for 5 surround-adaptation luminances.**

The data of Lingelbach and Haberich (1977) were approximated by Richter (1993) with a loudness formula ([1] in the figure 4) of Zwicker and Feldkeller (1967).



**Figure 5 Relative lightness and contrast of LABJND, CIELAB and CIEDE2000**

Figure 4 calculates the relative CIELAB contrast  $(L/dL)/(L/dL)_u$  from the relative CIELAB lightness  $(L^*/L_u^*)$ . In the  $L>L_u$  range the CIELAB contrast is larger than the LABJND contrast. The CIEDE2000 and LABJND contrast are more common for  $L>L_u$ . In the  $L<L_u$  range, the LABJND contrast is greater than the CIEDE2000 contrast.

Previous *one-step* CIELAB colorimetry:

The derivative of the CIELAB brightness  $L^*$  gives the luminance difference  $dL$ :

$$L^* = 116 (L/L_n)^{(1/3)} - 16 \quad (L \text{ is proportional to } Y \text{ and is used here.})$$

It follows for  $dL^*=1$

$$dL = (3/116) (L/L_u)^{2/3}$$

Result: The luminance difference  $dL_{\text{CIELAB}}$  calculated from CIELAB, and the visual psychophysical visual luminance difference  $dL_{\text{CIEDE2000}}$  of the CIEDE2000 experiments differ. (This is also why CIEDE2000 was developed without a color space).

As a result of this work, a *two-step* colorimetry seems necessary.

The success of CIELAB in many applications is undisputed. However, the CIELAB lightness  $L^*$  is not S-shaped, like the receptor responses in Figure 1. CIELAB is therefore increasingly faulty for luminance ranges with application contrasts  $C>25:1$ .

## Possible solutions to the problems

### 1. *Inductive colorimetry* by mathematical integration:

The integration of the visual contrast ( $L/dL$ ) results in the signal excitation  $F^*$ .

The integration of the luminance threshold  $dL$  gives the function  $L^*$ , the visual and colorimetric meaning of which has yet to be clarified, compare the *deductive colorimetry*.

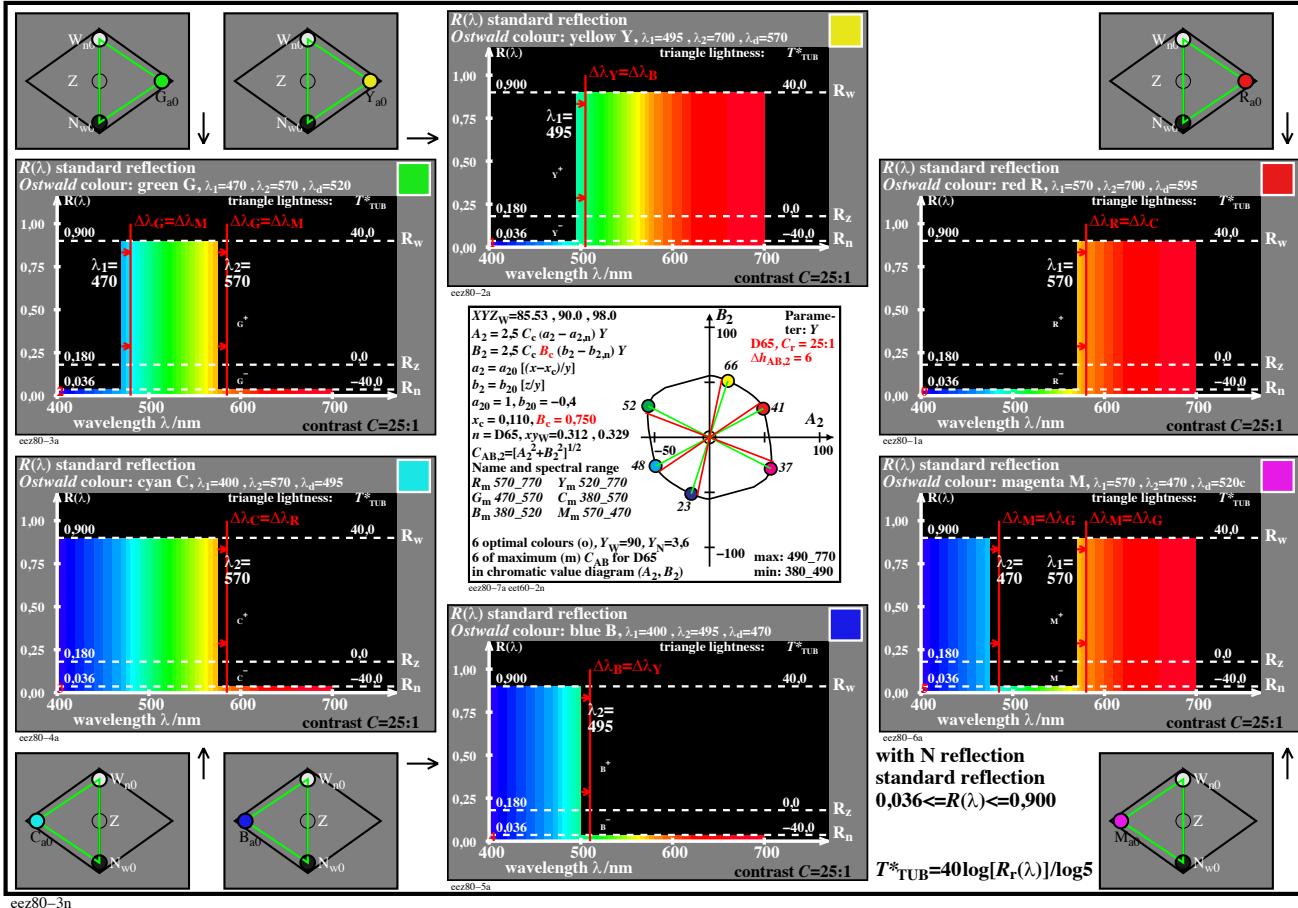
*Psychophysical* experiments give the visual luminance threshold  $dL$ .

### 2. *Deductive colorimetry* by mathematical derivation:

The derivative of the signal excitation  $F^*$  gives the contrast ( $L/dL$ ).

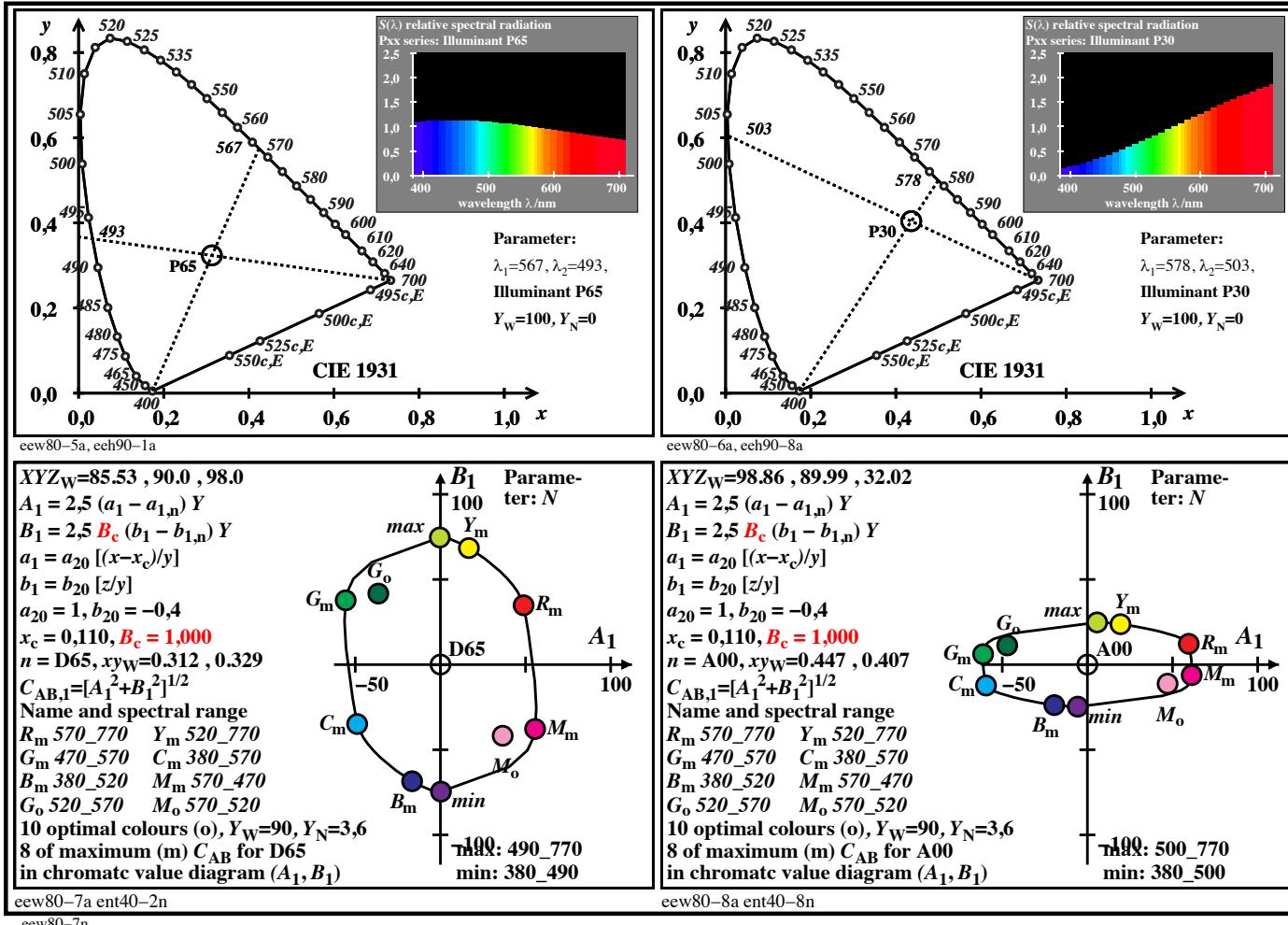
The derivative of the function  $L^*$  gives the luminance difference  $dL$ .

The color properties as a function of chromatic adaptation also require a *two-stage* colorimetry. The starting point is *psychophysical* experiments on hue thresholds of complementary optimal colors by *Holtsmark and Valberg* (1969), see next picture.

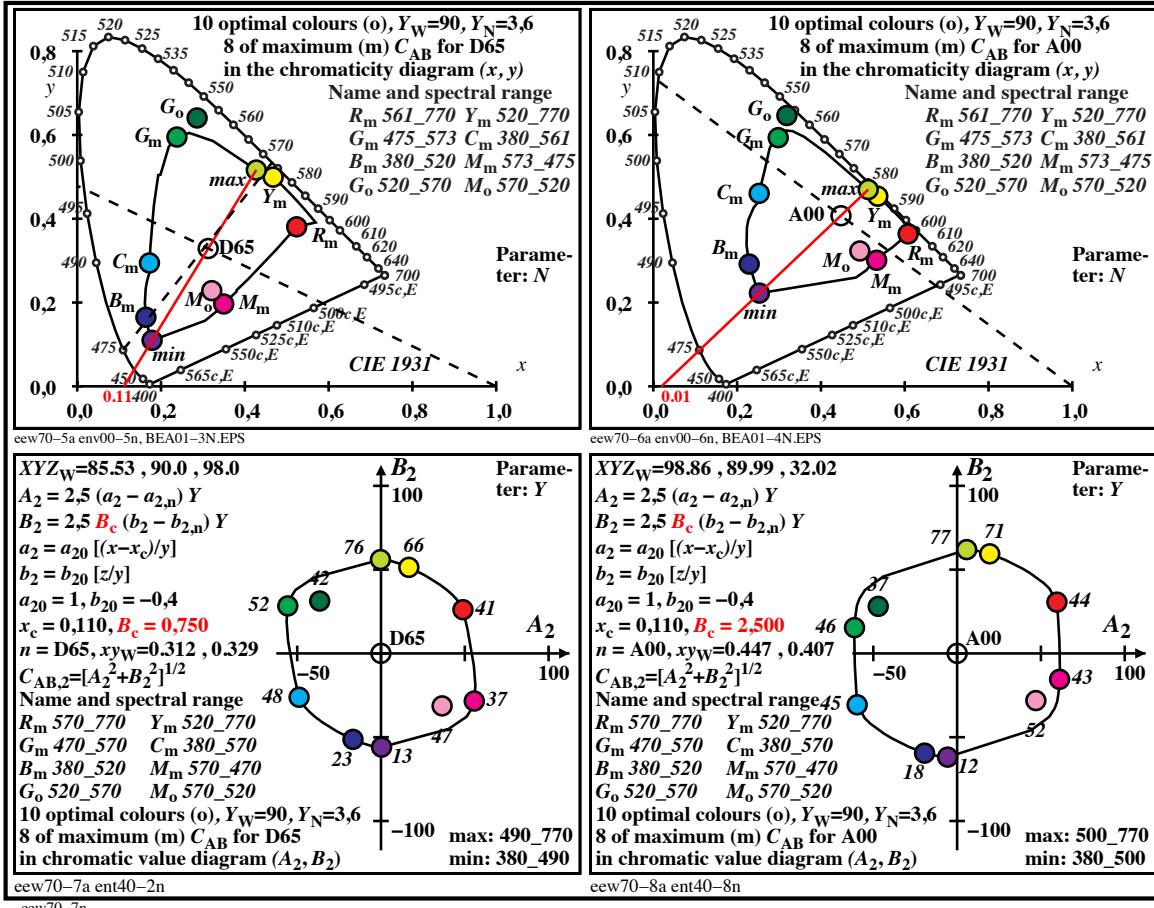


**Figure 6: Equal hue differentiation of complementary optimal colors**

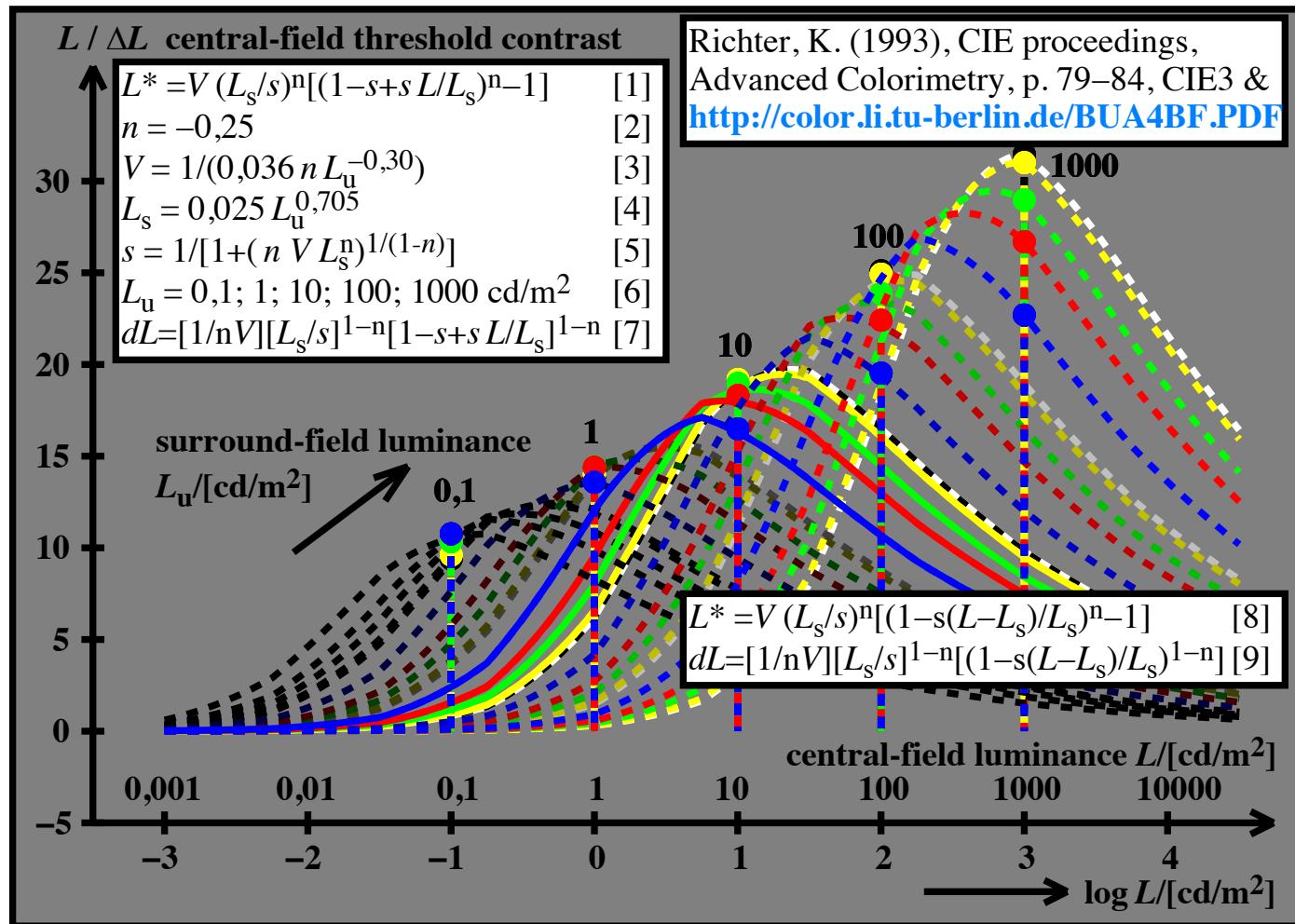
Spectral colors are mixed to optimal colors (top right). For example the hue differentiation of red and the complementary color cyan is the same, see arrows. The chromatic values  $C_{AB,2}$  and the hue-angle difference  $dh_{AB,2}$  are the same.



**Figure 7 For the standard illuminants D65 and A, the chromatic values  $C_{AB,1}$  are different. The constant  $B_c$  describes chromatic adaptation**



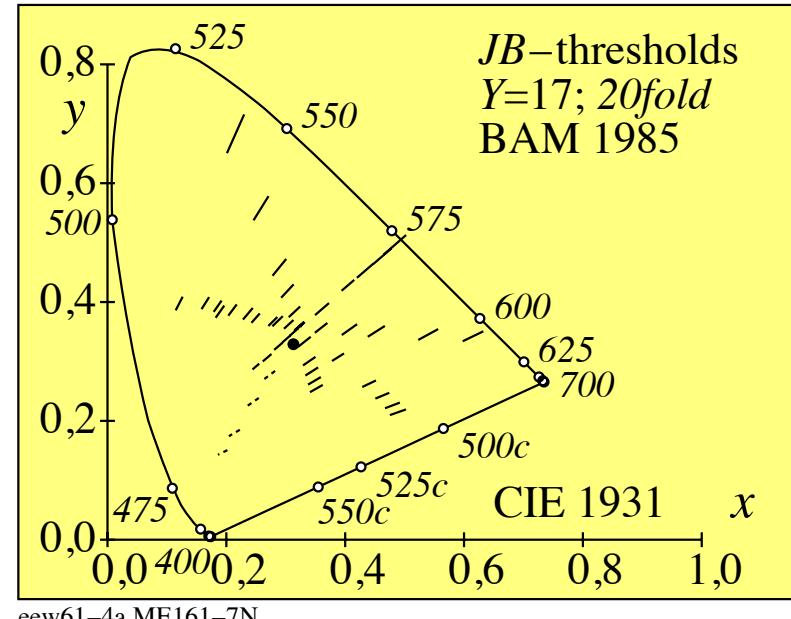
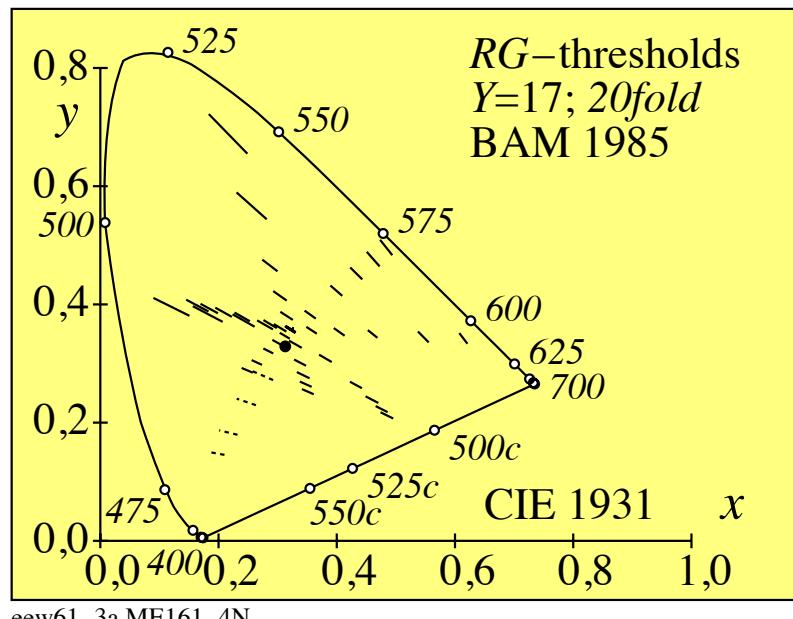
**Figure 8 Yellow (max) and blue (min) define chromatic value coordinates  $A_2$  and  $B_2$**   
For 3.6% black reflection, the radial chromatic values  $C_{AB,2}$  and also the ratios  $Y_E:Y_G$  of Eigen color (E) and Gegen color (G) are approximately the same.  
The luminous values  $Y$  of yellow, green, red, and blue serve for the reduction of the contrast ( $L/dL$ ) for chromatic colors compared to the achromatic colors in Figure 9.



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**Figure 9: Contrast ( $L/dL$ ) of achromatic and chromatic colors as function of chromatic and luminance adaptation**

For example for the surround luminance of  $100 \text{ cd/m}^2$ , the contrast ( $L/dL$ ) decreases from achromatic colours to yellow, green, red, and blue. Since  $L$  is constant for this adaptation luminance, the luminance difference  $dL$  and the radial chromaticity difference  $dc_{AB,2}$  increases. This radial increase is known from many psychophysical experiments.



**Figure 10: Radial increase in chromaticity difference  $dc_{xy}$  for psychophysical color thresholds in red-green and yellow-blue direction according to Richter (1985)**  
 Similar experimental results are known, for example, from MacAdam (1941) and the Munsell-color system (Newhall *et al.*, 1943). The colorimetric cause of this radial threshold increase is clearly visible in Figure 9.

## Summary

The deductive colorimetry uses *physiological* responses in monkeys according to *Valeton & van Norren* (1983) and describes the luminance signals  $F^*$ .

Inductive colorimetry uses psychophysical luminance thresholds. from *Lingelbach and Haberich* (1977) as well as the thresholds of Ostwald-optimal colors from *Holtsmark and Valberg* (1969). This allows to consider luminance and chromatic adaptation, which is characterized by only one parameter  $B_c$ .

The merge results in a *two-stage* colorimetry. The visual threshold  $dL$  allows the calculation of the excitation  $F^*$  and its differences  $dF^*$  by integrating the contrast ( $L/dL$ ). The excitation  $F^*$  can therefore not be determined by integration of  $dL$ , but only by integration of  $(L/dL)$ .

1. Therefore, the derivative  $dL_{\text{CIELAB}}$  of the lightness  $L^*_{\text{CIELAB}}$  cannot coincide with the visual difference  $dL_{\text{CIEDE2000}}$ .
2. Also, the integration of  $dL_{\text{CIEDE2000}}$  cannot match the lightness  $L^*_{\text{CIELAB}}$ .

The *two-stage* TUB colorimetry may replace the single-stage CIELAB and CIEDE2000 colorimetry. This may be a solution to the existing problems 1 and 2 and may produce a colour space for LABJND\_PF.

## Literature

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- Acknowledgements: I would like to thank Prof. Florian Süßl (BHT Berlin), Detlef Ruschin (HHI Berlin) and Thorstein Seim (University of Oslo) for their contributions to improving this work. Further parts of the *TUB-relativity model of colour vision for light and surface colours* are in preparation, see under publications: <http://color.li.tu-berlin.de/XY91FEN.html>

## Appendix

Stiles describes a general colour scaling function  $F$  as function of the receptor responses. In former times the receptor responses were named by the three colour vision deficiencies Protanop (P), Deuteranop (D), and Tritanop (T). One receptor type is missing for these observers. CIE has named the three receptors LMS instead of PDT. In the next figure, therefore, LMS with an index PDT is used.

**Colour-line element of Stiles  
(1946) with „colour values”  $L_P, M_D, S_T$   
three separate colour-response functions**

$$F(L_P) = i \ln(1 + 9L_P)$$

$$F(M_D) = j \ln(1 + 9M_D)$$

$$F(S_T) = k \ln(1 + 9S_T)$$

**Taylor-derivations:**

$$\begin{aligned}\Delta F(L_P, M_D, S_T) &= \frac{dF}{dL_P} \Delta L_P + \frac{dF}{dM_D} \Delta M_D + \frac{dF}{dS_T} \Delta S_T \\ &= \frac{9i}{1 + 9L_P} \Delta L_P + \frac{9j}{1 + 9M_D} \Delta M_D + \frac{9k}{1 + 9S_T} \Delta S_T\end{aligned}$$

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Figure 11: Line element of colour vision of Stiles connecting thresholds and scaling data

## Deductive and inductive antagonistic TUB colorimetry to improve CIE colorimetry for wide ranges of luminance and chromatic mapping

For a summary, see [http://color.li.tu-berlin.de/dfwg\\_23e.pdf](http://color.li.tu-berlin.de/dfwg_23e.pdf) or [http://color.li.tu-berlin.de/dfwg\\_23d.pdf](http://color.li.tu-berlin.de/dfwg_23d.pdf) in German.

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**Introduction:** ISO/CIE colorimetry, e.g. from CIELAB, CIELUV and CIEDE2000 according to ISO/CIE 11664-2, 5 and 4, is applied to surface paints with the light reference value range  $Y_N = 2.5 \leq Y \leq 90 = Y_W$ , see ISO/IEC 15775:2022. The ratio  $Y_w/Y_N = 36$  covers 1.5 logarithmic units. At the standard illuminance of 500 lux in offices, this corresponds to the luminance range  $4 \text{ cd/m}^2 \leq L \leq 142 \text{ cd/m}^2$  with the grey surround luminance  $L_u = 28 \text{ cd/m}^2$ .

The following TUBLAB-colorimetry is intended for the surround luminance range  $10 \leq L_u \leq 10000$ . The surround luminance ratio is 1000:1 and the sample luminance range is further increased by a factor of 36. TUB colorimetry is based on both *physiological* and *psychophysical* experimental data of color vision. For about five logarithmic units of the luminance range, physiological data of visual excitations and physiological data of luminance thresholds are analyzed. A TUB model for all achromatic and chromatic colors contains the antagonistic properties of the complementary *Ostwald* colors.

### Deductive and inductive TUB colorimetry for a wide luminancerange

The *deductive* TUB colorimetry begins with *physiological* data, e.g. from *Valeton and Van Norren (1973)*, which are approximated here by the function  $\tanh(x)$  (*tangens hyperbolicus*). All receptor-excitation functions, see <http://color.li.tu-berlin.de/eeg0/eeg00-5n.pdf> are S-shaped and similar. The derivative of an excitation function is *Gaussian-shaped*, see <http://color.li.tu-berlin.de/egg0/egg00-3n.pdf>

The *Inductive* TUB colorimetry begins with *psychophysical* data, e.g. of *Lingelbach and Haberich (1977)*, which were approximated by *Richter (1993)*, e.g. for the luminance threshold  $dL$  as a function of  $L$  and  $L_u$ , see <http://color.li.tu-berlin.de/egr4/egr40-1a.pdf>. The ratio  $L/dL$  is called the contrast and is *Gaussian-shaped*, see <http://color.li.tu-berlin.de/egr4/egr40-2a.pdf>. The *physiological* and *psychophysical* contrast functions coincide.

### Physiological and psychophysical contrast calculation and interpretation

Only for an adaptation luminance near  $100 \text{ cd/m}^2$ , the maximum contrast is near the adaptation luminance  $L_u$ . This is consistent with the physiological data (1973) of monkeys. The symmetry of the physiological contrast disappears in the psychophysical data. However, with short viewing times ( $<0.1\text{s}$ ) of the two adjacent sample luminances, the psychophysical contrast is symmetrical. Therefore, there is wide *agreement* of *deductive* and *inductive* contrast as a function of the luminances  $L$  and  $L_u$ .

### TUB colorimetry for a wide range of chromatic adaptation

*Richter (2020)* investigated the colorimetric properties of the complementary *Ostwald* optimal colors. For color values, wavelength limits, chromaticity and chromatic values for standard illuminant D50, see <http://color.li.tu-berlin.de/egh3/egh3.htm>. All *Ostwald* colours ( $o$ ) of a colour half have different luminous values  $Y_o$ , see <http://color.li.tu-berlin.de/eeg8/eeg81-5n.pdf>. However, the two chromatic values  $C_{AB,2}$  and  $C_{AB,3}$  give approximately the same radial chromatic value for each hue and illuminant. See <http://color.li.tu-berlin.de/eeg8/eeg81-7n.pdf> for the CIE illuminant D50 as an example.

### TUBLAB colorimetry for a wide range of luminance and chromaticity adaptations.

The antagonistic model TUBLAB for color vision can therefore be applied to a wide range of luminance and chromatic adaptation, for example instead of CIECAM16