## Hyperbolic response function of achromatic vision space $T^*_{HYP3X}$

| nonlinear color terms | name and relationship with test field luminance $L$ | notes |
|---|---|---|
| **threshold sum $T^*_{HYP3X}$ $T^*_{HYP3}$** | $T^*_{HYP3X} = A / (1 + A_2 / L^t)$; $X = 1/L^t = L^{-t}$<br>$= A_1 / (1 + A_2 \cdot X)$; $dX/dL = -t \cdot L^{-t-1}$<br>$= A_1 \cdot [1 + A_2 \cdot X)]^{-1}$ | equation type 1/(1+X)<br>*T. Seim* 2014:<br>exponent: $t$ = 0,8<br>for presentation time:<br>$t_p$ = 0,1s<br>of *Avramopoulos* experiments 1989<br>Hyperbolic function:<br>$T^*_{max} = A_1$<br>$T^*_{average} = 0{,}5 \cdot A_1$<br>$A_{2x} = A_2^{1/t}$ |
| **CIE luminance contrast sensitivity threshold $L$ / $dL$** | $dT^*_{HYP3X}/dX = -A_1 \cdot A_2 \cdot [1 + A_2 \cdot X)]^{-2}$<br>$dT^*_{HYP3X}/dL = dT^*_{HYP3X}/dX \cdot dX / dL$<br>$dT^*_{HYP3X}/dL = A_1 \cdot A_2 [1+A_2 \cdot X)]^{-2} t \cdot L^{-t-1}$<br>*for* $dT^*_{HYP3X}$=1, and multiplication with $L$:<br>$L / dL = A_1 \cdot A_2 [1+A_2 \cdot X)]^{-2} t \cdot L^{-t}$<br>$= A_1 \cdot A_2 \cdot t \cdot / ( L^t [1+A_2 / L^t)]^2 )$ | |
| **CIE luminance difference threshold $dL$** | $dL = L \cdot ( L^t [1+A_2 / L^t)] / (A_1 \cdot A_2 \cdot t)$<br>$= ( L^t + A_2)^2 / ( A_1 \cdot A_2 \cdot t \cdot L^{t-1} )$ | |

1−000030−L0 UE150−3N

## Logarithmic response function of achromatic vision space $T^*_{LOG3}$

| nonlinear color terms | name and relationship with test field luminance $L$ | notes |
|---|---|---|
| **threshold sum $T^*_{LOG3}$** | $T^*_{LOG3} = A_1 \cdot \log ( 1 + A_3 \cdot L )^t$<br>$= A_1 \cdot t \cdot \log ( X )$<br>$X = 1 + A_3 \cdot L$; $dX/dL = A_3$ | exponent: $t = A_2$ |
| **CIE luminance contrast sensitivity threshold $L$ / $dL$** | $dT^*_{LOG3} / dX = A_1 \cdot t \cdot X^{-1}$<br>$dT^*_{LOG3} / dL = dT^*_{LOG3} / dX \cdot dX / dL$<br>$dT^*_{LOG3} / dL = A_1 \cdot A_3 \cdot t \cdot X^{-1}$<br>*for* $dT^*_{LOG3}$=1, and multiplication with $L$:<br>$L / dL = L \cdot A_1 \cdot A_3 \cdot t \cdot X^{-1}$<br>$= L \cdot A_1 \cdot A_3 \cdot t \cdot ( 1 + A_3 \cdot L)^{-1}$ | for large $L$:<br>$T^*_{LOG3}$<br>$= A_1 \cdot t \cdot \log(A_3 \cdot L)$<br>for least square fit:<br>$dX/dA3 = 1$<br>$dX/dL = A_3$ |
| **CIE luminance difference threshold $dL$** | $dL = X / [ A_1 \cdot A_3 \cdot t ]$<br>$= [ 1 + A_3 \cdot L] / [ A_1 \cdot A_3 \cdot t ]$ | |

1−000030−L0 UE151−3N

## Exponential response function of achromatic vision space $T^*_{EXP3X}$

| nonlinear color terms | name and relationship with test field luminance $L$ | notes |
|---|---|---|
| **threshold sum $T^*_{EXP3X}$** | $T^*_{EXP3X} = A_1 \cdot \log (1+A_2 \cdot 10^{X-Xu})$; $X = \log L$<br>$= A_1 \cdot \log (1+A_2 \cdot 10^X / 10^{Xu})$; $L = 10^X$<br>$= A_1 \cdot \log (1+A_4 \cdot L)$; $dL/dX = \ln(10) \cdot 10^X$ | $L/dL$<br>$= A_1 \cdot A_4 \cdot L / (1+A_4 \cdot L)$<br>for large $L$ it is valid:<br>$1 << A_4 \cdot L$<br>therefore:<br>$L/dL = A_1$<br>= constant<br>=*Weber−Fechner* law<br>$A_4 = A_2 / L_u$ |
| **CIE luminance contrast sensitivity threshold $L$ / $dL$** | $dT^*_{EXP3X}/dL = A_1 \cdot A_4 / ( 1 + A_4 \cdot L)$<br>$= A_1 \cdot A_2 / [L_u \cdot (1+A_2 \cdot (L/L_u)]$<br>for $dT^*_{EXP3X}$=1, and multiplication with $L$:<br>$L / dL = A_1 \cdot A_4 \cdot L / ( 1 + A_4 \cdot L)$<br>$= A_1 \cdot A_2 \cdot (L/L_u) \cdot [1 + A_2 \cdot (L/L_u)]$<br>The ratio $L$ / $dL$ is constant for large $L$ | |
| **CIE luminance difference threshold $dL$** | $dL = ( 1 + A_4 \cdot L) / (A_1 \cdot A_4)$<br>$= [ 1 + A_2 \cdot (L/L_u)] / (A_1 \cdot A_2/L_u)$ | |

1−000030−L0 UE150−7N

## Logarithmic response function of achromatic vision space $T^*_{LOG3}$

| nonlinear color terms | name and relationship with test field luminance $L$ | notes |
|---|---|---|
| **threshold sum $T^*_{LOG3}$** | $T^*_{LOG3} = A_1 \cdot \log ( 1 + A_2 \cdot L + A_3 \cdot L^2 )$<br>$= A_1 \log ( X )$<br>$X = 1+A_2 \cdot L+A_3 \cdot L^2$; $dX/dL = A_2+A_3 \cdot L$ | for large $L$:<br>$T^*_{LOG3}$<br>$= A_1 \cdot \log(A_3 \cdot L^2)$ |
| **CIE luminance contrast sensitivity threshold $L$ / $dL$** | $dT^*_{LOG3} / dX = A_1 \cdot X^{-1}$<br>$dT^*_{LOG3} / dL = dT^*_{LOG3} / dX \cdot dX / dL$<br>$dT^*_{LOG3} / dL = A_1 \cdot (A_2 + A_3 \cdot L) \cdot X^{-1}$<br>*for* $dT^*_{LOG3}$=1, and multiplication with $L$:<br>$L / dL = L \cdot A_1 \cdot (A_2 + A_3 \cdot L) \cdot X^{-1}$<br>$= L \cdot A_1 (A_2+A_3 \cdot L)/(1+A_2 L+A_3 \cdot L^2)$ | for least square fit:<br>$dX/dA2 = L$<br>$dX/dA3 = L^2$<br>$dX/dL = A_2 + 2A_3 \cdot L$ |
| **CIE luminance difference threshold $dL$** | $dL = X / [A_1 \cdot (A_2 + A_3 \cdot L) ]$<br>$= (1+A_2 \cdot L+A_3 \cdot L^2) / [A_1 (A_2+A_3 \cdot L)]$ | |

1−000030−L0 UE151−7N

TUB-test chart UE15; Colour thresholds spaces
Hyperbolic and logarithmic luminance sensitivity

input: *w/rgb/cmyk −> w/rgb/cmyk−*
output: no change