Color threshold formula LABJNDS 1985 for *NW* achromatic colours

$$\Delta E^*_{JND,NW} = Y_0 [(\Delta Y_W)^2 + (\Delta c_{ab,W} \cdot Y_W)^2]^{1/2} / (s + q \cdot Y_W{}^t)$$
$$= Y_0 [(\Delta Y_N)^2 + (\Delta c_{ab,N} \cdot Y_N)^2]^{1/2} / (s + q \cdot Y_N{}^t)$$

$a = x / y$ $\quad a_n = x_n / y_n$ $\quad b = -0{,}4\, z / y$ $\quad b_n = -0{,}4\, z_n / y_n$

$c_{ab} = [a_0{}^2 (a - a_n)^2 + b_0{}^2 (b - b_n)^2]^{1/2}$ $\quad$ n = D65 or A (surround)

$Y = (Y_1 + Y_2) / 2$ $\quad \Delta Y = Y_1 - Y_2$ $\quad \Delta a = a_1 - a_2$ $\quad \Delta b = b_1 - b_2$

$p_{c,NW} = c_{ab} / c_{ab,NW}$ $\quad$ s = 0,0170 $\quad$ q = 0,0058 $\quad$ t = 1,0

$a_0 = 1{,}0$ $\quad b_0 = 1{,}8$ $\quad Y_0 = 1{,}5$ $\quad$ surround D65

$a_0 = 1{,}0$ $\quad b_0 = 1{,}7$ $\quad Y_0 = 1{,}0$ $\quad$ surround A

Just noticeable difference of complementary (c) *NW* colours with:

$(a_W - a_n)Y_W=(a_N - a_n)Y_N$; $\quad (b_W - b_n)Y_W=(b_N - b_n)Y_N$; $\quad c_{ab,W}Y_W=c_{ab,N}Y_N$

$\Delta Y_W = \text{const} (s + q \cdot Y_W{}^t) / Y_0$ $\quad$ in luminance direction *WN*

$\Delta c_{ab,W} \cdot Y_W = \text{const} (s + q \cdot Y_W{}^t) / Y_0$

$\Delta c_{ab,N} \cdot Y_N = \text{const} (s + q \cdot Y_N{}^t) / Y_0$ $\quad$ in any chromaticity direction c_{ab} and for the *NW* purity $p_{c,NW}$=0

1−000030−L0 $\quad$ UE110−3N

Color threshold formula LABJNDS 1985 for *Ostwald* (o) colours

$$\Delta E^*_{JND,o} = Y_0 [(\Delta Y_o)^2 + (a_0 \Delta a_o \cdot Y_o)^2 + (b_0 \Delta b_o \cdot Y_o)^2]^{1/2} / (s + q \cdot Y_o{}^t)$$
$$= Y_0 [(\Delta Y_o)^2 + (\Delta c_{ab,o} \cdot Y_o)^2]^{1/2} / (s + q \cdot Y_o{}^t)$$

$a = x / y$ $\quad a_n = x_n / y_n$ $\quad b = -0{,}4\, z / y$ $\quad b_n = -0{,}4\, z_n / y_n$

$c_{ab} = [a_0{}^2 (a - a_n)^2 + b_0{}^2 (b - b_n)^2]^{1/2}$ $\quad$ n = D65 or A (surround)

$Y = (Y_1 + Y_2) / 2$ $\quad \Delta Y = Y_1 - Y_2$ $\quad \Delta a = a_1 - a_2$ $\quad \Delta b = b_1 - b_2$

$p_{c,o} = c_{ab} / c_{ab,o}$ $\quad$ s = 0,0170 $\quad$ q = 0,0058 $\quad$ t = 1,0

$a_0 = 1{,}0$ $\quad b_0 = 1{,}8$ $\quad Y_0 = 1{,}5$ $\quad$ surround D65

$a_0 = 1{,}0$ $\quad b_0 = 1{,}7$ $\quad Y_0 = 1{,}0$ $\quad$ surround A

Just noticeable difference of complementary (c) *Ostwald* (o) colours with:

$(a_o - a_n)Y_o=(a_{oc} - a_n)Y_{oc}$; $\quad (b_o - b_n)Y_o=(b_{oc} - b_n)Y_{oc}$; $\quad c_{ab,o}Y_o=c_{ab,oc}Y_{oc}$

$\Delta Y_o = \text{const} (s + q \cdot Y_o{}^t) / Y_0$ $\quad$ in luminance direction *WN*

$\Delta c_{ab,o} \cdot Y_o = \text{const} (s + q \cdot Y_o{}^t) / Y_0$

$\Delta c_{ab,oc} \cdot Y_{oc} = \text{const} (s + q \cdot Y_{oc}{}^t) / Y_0$ $\quad$ in any chromaticity direction c_{ab} and for the *Ostwald* purity $p_{c,o}$=1

1−000030−L0 $\quad$ UE110−7N

Complementary optimal colours: Relation *XYZ* and chromatic values *A, B*

nonlinear color terms	name and relationship with tristimulues values *XYZ* , and the chromatic values *(A, B)*	notes
Threshold space *ABY−JND1* equation (1)	$\Delta E^*_{ABY} = Y_0\{[a_0\Delta \boldsymbol{A}_{01}]^2 + [b_0\Delta \boldsymbol{B}_{01}]^2 + [\Delta \boldsymbol{Y}_{01}]^2\}^{1/2} = Y_0\{[c_0\Delta \boldsymbol{C}_{ab,01}]^2 + [\Delta \boldsymbol{Y}_{01}]^2\}^{1/2}$	$A=(a-a_n)\cdot Y = (x/y - x_n/y_n)\cdot Y$ Normalization similar to CIELAB: $X_{01} = X/X_n$; $Y_{01} = Y/Y_n$; $Z_{01} = Z/Z_n$ Relation for complementary (c) colours: $X_{01c}=1-X_{01}$; $Y_{01c}=1-Y_{01}$ $Z_{01c}=1-Z_{01}$ Chromatic values: $A_{01}=(a_{01}-a_{01n})\cdot Y_{01} = (x_{01}/y_{01} - 1)\cdot Y_{01} = (X_{01}/Y_{01} - 1)\cdot Y_{01} = X_{01} - Y_{01} = -A_{01c}$
Threshold space *ABY−JND2* equation (2)	$\Delta E^*_{ABY} = Y_0\{[a_0\Delta \boldsymbol{A}_{01}]^2 + [b_0\Delta \boldsymbol{B}_{01}]^2 + [\Delta \boldsymbol{Y}_{01} / \boldsymbol{Y}_{01}]^2\}^{1/2} = Y_0\{[c_0\Delta \boldsymbol{C}_{ab,01}]^2 + [\Delta \boldsymbol{Y}_{01} / \boldsymbol{Y}_{01}]^2\}^{1/2}$	
Threshold space *ABY−JND3* equation (3)	$\Delta E^*_{ABY} = Y_0\{[a_0\Delta \boldsymbol{A}_{01}/\boldsymbol{A}_{01}]^2 + [b_0\Delta \boldsymbol{B}_{01}/\boldsymbol{B}_{01}]^2 + [\Delta \boldsymbol{Y}_{01}/\boldsymbol{Y}_{01}]^2\}^{1/2} = Y_0\{[c_0\Delta \boldsymbol{C}_{ab,01}/\boldsymbol{C}_{ab,01}]^2 + [\Delta \boldsymbol{Y}_{01}/\boldsymbol{Y}_{01}]^2\}^{1/2}$	
Properties complementary colours	$a_{01c}=-a_{01}$; $b_{01c}=-b_{01}$; $C_{ab,01c}=C_{ab,01}$; $\Delta a_{01c}=\Delta a_{01}$; $\Delta b_{01c}=\Delta b_{01}$; $\Delta C_{ab,01c}=\Delta C_{ab,01}$; $\Delta Y/Y$=const	

1−000030−L0 $\quad$ UE111−3N

Complementary optimal colours: Relation *XYZ* and chromatic values *A, B*

nonlinear color terms	name and relationship with tristimulues values *XYZ*, and the chromatic values *(A, B)*	notes
Threshold space *ABY−JND1* equation (1)	$\Delta E^*_{ABY} = Y_0\{[a_0\Delta \boldsymbol{A}_{01}]^2 + [b_0\Delta \boldsymbol{B}_{01}]^2 + [\Delta \boldsymbol{Y}_{01}]^2\}^{1/2} = Y_0\{[c_0\Delta \boldsymbol{C}_{ab,01}]^2 + [\Delta \boldsymbol{Y}_{01}]^2\}^{1/2}$	$A=(a-a_n)\cdot Y = (x/y - x_n/y_n)\cdot Y$ Normalization similar to CIELAB: $X_{01} = X/X_n$; $Y_{01} = Y/Y_n$; $Z_{01} = Z/Z_n$ Relation for complementary (c) colours: $X_{01c}=1-X_{01}$; $Y_{01c}=1-Y_{01}$ $Z_{01c}=1-Z_{01}$ Chromatic values: $A_{01}=(a_{01}-a_{01n})\cdot Y_{01} = (x_{01}/y_{01} - 1)\cdot Y_{01} = (X_{01}/Y_{01} - 1)\cdot Y_{01} = X_{01} - Y_{01} = -A_{01c}$ c=0,017; d=0,580
Threshold space *ABY−JND4* equation (4)	$\Delta E^*_{ABY} = Y_0\{[a_0\Delta \boldsymbol{A}_{01}/\boldsymbol{Y}_{01}]^2 + [b_0\Delta \boldsymbol{B}_{01}/\boldsymbol{Y}_{01}]^2 + [\Delta \boldsymbol{Y}_{01}/\boldsymbol{Y}_{01}]^2\}^{1/2} = Y_0\{[c_0\Delta \boldsymbol{C}_{ab,01}/\boldsymbol{Y}_{01}]^2 + [\Delta \boldsymbol{Y}_{01}/\boldsymbol{Y}_{01}]^2\}^{1/2}$	
Threshold space *ABY−JND5* equation (5)	$\Delta E^*_{ABY} = Y_0\{[a_0\Delta \boldsymbol{a}_{01}\cdot \boldsymbol{Y}_{01}]^2 + [b_0\Delta \boldsymbol{b}_{01}\cdot \boldsymbol{Y}_{01}]^2 + [\Delta \boldsymbol{Y}_{01}]^2\}^{1/2}/(c+d\cdot \boldsymbol{Y}_{01}) = Y_0\{[c_0\Delta \boldsymbol{c}_{ab,01}\cdot \boldsymbol{Y}_{01}]^2 + [\Delta \boldsymbol{Y}_{01}]^2\}^{1/2}/(c+d\cdot \boldsymbol{Y}_{01})$	
Properties complementary colours	$a_{01c}=-a_{01}$; $b_{01c}=-b_{01}$; $C_{ab,01c}=C_{ab,01}$; $\Delta a_{01c}=\Delta a_{01}$; $\Delta b_{01c}=\Delta b_{01}$; $\Delta C_{ab,01c}=\Delta C_{ab,01}$;	

1−000030−L0 $\quad$ UE111−7N

TUB-test chart UE11; Colour threshold spaces *LABJND & ABYJND*, and modifications

input: *w/rgb/cmyk −> w/rgb/cmyk*−
output: no change