## Color threshold formula LABJNDS 1985 (JND=just noticeable difference)

$\Delta E^*_{JND} = Y_0 [ (\Delta Y)^2 + ( a_0 \Delta a'' \cdot Y)^2 + ( b_0 \Delta b'' \cdot Y)^2 ]^{1/2} / ( s + q \cdot Y^g )$

$a = x / y \quad a_n = x_n / y_n \quad b = -0{,}4\, z / y \quad b_n = -0{,}4\, z_n / y_n$

$a'' = a_n + ( a - a_n) / ( 1 + 0{,}5 \, | a - a_n | )$

$b'' = b_n + ( b - b_n) / ( 1 + 0{,}5 \, | b - b_n | )$

n = D65 or A (surround)

$Y = ( Y_1 + Y_2 ) / 2 \quad \Delta Y = Y_1 - Y_2 \quad \Delta a'' = a''_1 - a''_2 \quad \Delta b'' = b''_1 - b''_2$

$s = 0{,}0170 \quad q = 0{,}0058 \quad g = 1{,}0$

$a_0 = 1{,}0 \quad b_0 = 1{,}8 \quad Y_0 = 1{,}5$ surround D65

$a_0 = 1{,}0 \quad b_0 = 1{,}7 \quad Y_0 = 1{,}0$ surround A

**Just noticeable difference (JND) in four colour directions**

| | |
|---|---|
| $\Delta Y = \text{const} ( s + q \cdot Y^g ) / Y_0$ | in luminance direction *WN* |
| $\Delta a'' \cdot Y = \text{const} ( s + q \cdot Y^g ) / (Y_0 \cdot a_0)$ | in chromaticity direction *RG* |
| $\Delta b'' \cdot Y = \text{const} ( s + q \cdot Y^g ) / (Y_0 \cdot b_0)$ | in chromaticity direction *YB* |
| $\Delta c_{ab}'' \cdot Y = \text{const} ( s + q \cdot Y^g ) / (Y_0 \cdot [a_0^2 + b_0^2]^{1/2})$ | in any chromaticity direction $c_{ab}$ |

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## Colour spaces CIELAB 1976, sRGB, & elementary *Ostwald* space oRGB$_e$

| nonlinear color terms | name and relationship with tristimulues value *Y* | *notes* *g*= 1/2,4 = gamma |
|---|---|---|
| **lightness** CIELAB aLAB | $L^*_{CIELAB} = 116 ( Y / Y_n )^{1/3} - 16$ *(Y > 0,8)*<br>approximation: $L^*_{aLAB} = 100 (Y/Y_n)^g$ *(Y >0)* | CIELAB 1976<br>$Y_n = 100$ |
| **triangle brillance** sRGB$_d$, see IEC 61966-2-1 | $T^*_{sRGB,d} = 100 ( Y / Y_{0,d} )^g$ *(Y >0; 7<=$Y_{0,d}$<=100)*<br>$= 100 ( Y / Y_n )^g \cdot ( Y_n / Y_{0,d} )^g$<br>$= L^*_{aLAB} \cdot ( Y_n / Y_{0,d} )^g$ | sRGB$_d$, $Y_{0Wd} = 100$<br>$Y_{sRGBd} = 22, 71, 7$<br>$Y_{sCMYd} = 78, 29, 93$<br>*d* = device colours |
| **triangle brillance** oRGB$_e$, see CIE R1−57 | $T^*_{oRGB,e} = 100 ( Y / Y_{o,e} )^g$ *(Y >0; 7<=$Y_{o,e}$<=100)*<br>$= 100 ( Y / Y_n )^g \cdot ( Y_n / Y_{o,e} )^g$<br>$= L^*_{aLAB} \cdot ( Y_n / Y_{o,e} )^g$ | oRGB$_e$, $Y_{oWe} = 100$<br>$Y_{oRGBe} = 41, 62, 19$<br>$Y_{oCMYe} = 45, 26, 84$<br>*e* = elementary colours |
| **luminance difference threshold** $dT^*_{oRGB,e}=1$ | $dT^*_{oRGB,e} / dY = dL^*_{aLAB} / dY \cdot ( Y_n / Y_{o,e} )^g$<br>$= 100\, g\, ( Y / Y_n )^{g-1} \cdot ( Y_n / Y_{o,e} )^g$<br>$dY = \text{const} \cdot ( Y / Y_n )^{1-g} \cdot ( Y_{o,e} / Y_n )^g$ | *o* = optimal colours of *Ostwald* of maximum chromatic value $C_{ab}$ see CIE R1−57 |

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## Color threshold formula LABJNDS 1987 (JND=just noticeable difference)

| nonlinear color terms | name and relationship with tristimulues value *Y* and chromaticitity *(a, b)* | *notes* |
|---|---|---|
| ***WN*-response or scaling function** $T^*_L$ | **response in white−black direction**<br>$T^*_L = \pm A_1/A_7 \cdot \log(1+A_7 \cdot \lvert Y_{1,2} - Y_n \rvert \cdot F_Y)$<br>with $Y_1 = Y - 0{,}5 \cdot \Delta Y$<br>$Y_2 = Y + 0{,}5 \cdot \Delta Y$ | $F_Y = 1/(1+A_2 \cdot Y)$ is identical for the three responses<br>$Y_n$ tristimulues value of the surround (n)<br>$a = x/y$<br>$b = -0{,}4 \cdot (1-x-y)/y$<br>$c = \{ [a-a_n]^2 + [b-b_n]^2 \}^{1/2}$<br>$(a_n, b_n)$ chromaticity of the surround (n)<br>$A_1$=58, $A_2$=0,39, $A_3$=0,85, $A_4$=0,37, $A_5$=31, $A_6$=19, $A_7$=0,015<br>physical: **amplitude modulation** |
| ***RG*-response or scaling function** $T^*_A$ | **response in red−green direction**<br>$T^*_L = \pm A_5/A_3 \cdot \log(1+A_3 \cdot \lvert a_{1,2} - a_n \rvert \cdot Y \cdot F_Y)$<br>with $a_1 = a - 0{,}5 \cdot \Delta Y$<br>$a_2 = a + 0{,}5 \cdot \Delta Y$ | |
| ***YB*-response or scaling function** $T^*_B$ | **response in yellow−blue direction**<br>$T^*_L = \pm A_6/A_4 \cdot \log(1+A_4 \cdot \lvert b_{1,2} - b_n \rvert \cdot Y \cdot F_Y)$<br>with $b_1 = b - 0{,}5 \cdot \Delta Y$<br>$b_2 = b + 0{,}5 \cdot \Delta Y$ | |
| ***radial physiological* response or scaling function** $T^*_C$ | ***physiological* response in *radial* direction**<br>$T^*_C = \text{const} \cdot [\log(1 + A_2 \cdot Y + A_r \cdot \lvert c_{1,2} - c_n \rvert \cdot Y) - \log(1 + A_2 \cdot Y)$<br>$T^*_C$ = achromatic + chromatic response − achromatic response | |

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## Elementary colour space with *Ostwald* colours oRGB$_e$, and colours tRGB$_e$

| nonlinear color terms | name and relationship with tristimulues value *Y*, and the chromaticity values *(x,y)* | notes *g*= 1/2,4 = gamma |
|---|---|---|
| **triangle brillance** oRGB$_e$, see CIE R1−57 | $I^*_{oRGB,e} = 100 ( Y / Y_{o,e} )^g$ *(Y >0; 7<=$Y_{o,e}$<=100)*<br>$= 100 ( Y / Y_n )^g \cdot ( Y_n / Y_{o,e} )^g$<br>$= L^*_{aLAB} \cdot ( Y_n / Y_{o,e} )^g$ | oRGB$_e$, $Y_{oWe} = 100$<br>41,62,19<=$Y_{tRGBe}$<=100<br>45,26,84<=$Y_{tCMYe}$<=100<br>*e* = elementary colours |
| **triangle brillance** tRGB$_e$ of top color solid | $I^*_{tRGB,e} = 100 ( Y / Y_{t,e} )^g$ *(Y >0; 7<=$Y_{t,e}$<=100)*<br>$= 100 ( Y / Y_n )^g \cdot ( Y_n / Y_{t,e} )^g$<br>$= L^*_{aLAB} \cdot ( Y_n / [Y_n + p_{co}(Y_{oe} - Y_n)] )^g$ | *o* = optimal colours of *Ostwald* of maximum chromatic value $C_{ab}$, see CIE R1−57<br>*t* = colours on top of colour triangle<br>$p_{eo} = ( x - x_n)/( x_o - x_n)$ = rel. excitation purity<br>$p_{co} = p_{eo} \cdot y_o / y$<br>$p_{coa} = ( a - a_n)/( a_o - a_n)$ |
| **luminance difference threshold** $dI^*_{tRGB,e}=1$ | $dI^*_{tRGB,e} / dY = dL^*_{aLAB} / dY \cdot ( Y_n / Y_{t,e} )^g$<br>$= 100 g \cdot ( Y / Y_n )^{g-1} \cdot ( Y_n / Y_{t,e} )^g$<br>$dY = \text{const} \cdot (Y/Y_n)^{1-g} ([Y_n + p_{co}(Y_{oe} - Y_n)]/Y_n)^g$ | |
| **chromaticity difference threshold** | $dI^*_{tRGB,e} / da = L^*_{aLAB} \cdot d[(Y_n / Y_{t,e})^g] / da$<br>$da = L^*_{aLAB} ([Y_n + p_{coa}(Y_{oe} - Y_n)]/Y_n)^{g-1} [Y_{oe} - Y_n]/Y_n$ | |

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