Weber-Fechner law in CIE 230:2019 for threshold colour differences of surface colours

The *Weber-Fechner* law describes the lightness L^*_r as ***logarithmic*** function of L_r.
The *Stevens* law describes the lightness L^*_{CIELAB} as ***potential*** function of $L_r=Y/5$.
IEC 61966-2-1 uses a similar potential function $L^*_{IEC} = m\, L_r^{1/2,4}$.
The *Weber-Fechner* law is equivalent to the equation: $\Delta L_r = c\, L_r$ [1]
Integration leads to the logarithmic equation: $L^*_r = k \log(L_r)$. [2]
Derivation for $\Delta L^*_r=1$ leads to the linear equation: $L_r/\Delta L_r = k = 57$. [3]
For colours in offices the **standard contrast range** is **25:1**=90:3,6.

Table 1: CIE tristimulus value *Y*, luminance *L*, and lightnesses *L**

Colour (matte)	Tritimulus value	office luminance	relative luminance	CIE lightness	relative lightness
(contrast) **(25:1=90:3,6)**	***Y***	***L*** **[cd/m²]**	$L_r = L/L_u$	$L^*_{CIELAB} \sim m\, L_r^{1/2,4}$	$L^*_r = k \log(L_r)$
White W (paper)	90 =18*5	142 =28,2*5	5	94 =50+44	40 $=k\log(5)$
Grey Z (paper)	18	28,2	1	50	0 $=k\log(1)$
Black N (paper)	3,6 =18/5	5,6 28,2/5	0,2	18 50−32	−40 $=k\log(0,2)$

For the lightness range between $L^*_r=-40$ and 40 the constant is: $k=40/\log(5)=57$

EE540−1N

Weber-Fechner law in CIE 230:2019 for threshold colour differences of surface colours and two ranges $0,2 <= L_r <=1$ and $1 <= L_r <=5$

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IEC 61966-2-1 uses a similar potential function $L^*_{IEC} = m\, L_r^{1/2,4}$.
The *Weber-Fechner* law is equivalent to the linear equation: $\Delta L_r = c_i L_r$ $(i=0,1)$ [1]
Integration leads to the logarithmic equation: $L^*_r = k_i \log(L_r)$ $(i=0,1)$ [2]
Derivation leads for $\Delta L^*_r=1$ to the linear equation: $L_r/\Delta L_r = k_i$ ($k_0=46$, $k_1=63$) [3]
For colours in offices the **standard contrast range** is **25:1**=90:3,6.

Table 1: CIE tristimulus value *Y*, luminance *L*, and lightnesses *L**

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White W (paper)	90 =18*5	142 =28,2*5	5	94 =50+44	44 $=k_1\log(5)$
Grey Z (paper)	18	28,2	1	50	0 $=k_0\log(1)$
Black N (paper)	3,6 =18/5	5,6 28,2/5	0,2	18 50−32	−32 $=k_0\log(0,2)$

For the two lightness ranges it is $k_0=-32/\log(0,2)=46$ and $k_1=44/\log(5)=63$.

EE540−2N

line element of *Stiles* (1946) with „color values" L_P, M_D, S_T

three separate color signal functions

$$F(L_P) = i \ln(1+9L_P)$$
$$F(M_D) = j \ln(1+9M_D)$$
$$F(S_T) = k \ln(1+9S_T)$$

***Taylor*-derivations:**

$$\Delta F(L_P,M_D,S_T) = \frac{dF}{dL_P}\Delta L_P + \frac{dF}{dM_D}\Delta M_D + \frac{dF}{dS_T}\Delta S_T$$
$$= \frac{9i}{1+9L_P}\Delta L_P + \frac{9j}{1+9M_D}\Delta M_D + \frac{9k}{1+9S_T}\Delta S_T$$

EE541−1N

line element of *Vos&Walraven* (1972) with „color values" L_P, M_D, S_T

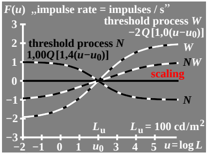
three separate color signal functions

$$F(L_P) = -2i\sqrt{L_P}$$
$$F(M_D) = -2j\sqrt{M_D}$$
$$F(S_T) = -2k\sqrt{S_T}$$

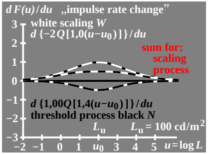
***Taylor*-derivations:**

$$\Delta F(L_P,M_D,S_T) = \frac{dF}{dL_P}\Delta L_P + \frac{dF}{dM_D}\Delta M_D + \frac{dF}{dS_T}\Delta S_T$$
$$\Delta F(L_P,M_D,S_T) = \frac{i}{\sqrt{L_P}}\Delta L_P + \frac{j}{\sqrt{M_D}}\Delta M_D + \frac{k}{\sqrt{S_T}}\Delta S_T$$

EE541−2N



EE540−3N



EE540−4N

functions $q[k(u-u_0)]$ „achromatic signal"-description

with $u = \log L$ (L = luminance)
$u_0 = \log L_u$ (L_u = surround luminance)

$$q[k(u-u_0)] = 1 + 1/[1+\sqrt{2}e^{k(u-u_0)}]$$

function values:

$q[k(u-u_0) \rightarrow +\infty] = 1$
$q[k(u-u_0) = 0] = \sqrt{2}$
$q[k(u-u_0) \rightarrow -\infty] = 2$

EE541−3N

„achromatic signal"-description functions $Q_{1m}[k(u-u_0)]$

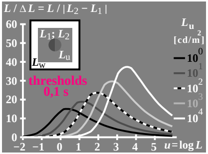
with $u = \log L$ (L = luminance)
$u_0 = \log L_u$ (L_u = surround luminance)

$$Q_{1m}[k(u-u_0)] = \frac{l}{\ln\sqrt{2}} \ln q[k(u-u_0)] - m$$

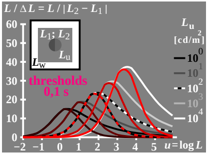
function values with $l = m = 1$:

$Q[k(u-u_0) \rightarrow +\infty] = 1$
$Q[k(u-u_0) = 0] = 0$
$Q[k(u-u_0) \rightarrow -\infty] = -1$

EE541−4N



EE540−5N



EE540−6N

„achromatic signal" discrimination as function of relative light density $h = \ln H = k(u-u_0)$, ln = natural log.

$$Q' = \frac{d}{dH}[\ln\{1+1/(1+\sqrt{2}H)\}]/\ln\sqrt{2}$$
$$= -\sqrt{2}/[\ln\sqrt{2}(1+\sqrt{2}H)(2+\sqrt{2}H)]$$

function values:

$Q'[k(u-u_0) \rightarrow +\infty] = 0$
$Q'[k(u-u_0) = 0] = -0,5$
$Q'[k(u-u_0) \rightarrow -\infty] = 0$

EE541−5N

luminance discrimination possibility $L/\Delta L$ as function of H

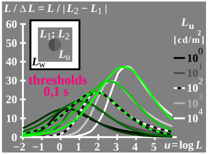
with: $L = 10^u$ $H = e^h = 10^{\log e\, k(u-u_0)}$
$dL/du = \ln 10\, L$ $dH/du = k\, H$

it follows: $L/\Delta L = [k H/(dH \ln 10)]$

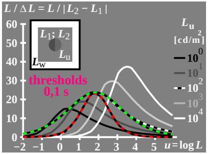
$$\frac{L}{dL} = \text{const } H/[(1+\sqrt{2}H)(2+\sqrt{2}H)]$$

$Q'[k(u-u_0) \rightarrow +\infty] = 0$
$Q'[k(u-u_0) = 0] =$ maximum
$Q'[k(u-u_0) \rightarrow -\infty] = 0$

EE541−6N



EE540−7N



EE540−8N

double line element of *Richter* (1987) for the lighting technology with the luminance $L = f(L_P, M_D, S_T)$

$$F(L) = \int_{-\infty}^{L} (L/\Delta L)\, dL \quad \text{(relative lightness?)}$$
$$F(L) = iQ(H) = \begin{cases} \underline{i}\, Q(\underline{H}) & (u<u_0) \\ \bar{i}\, Q(\bar{H}) & (u \ge u_0) \end{cases}$$

with: $\underline{k}=1,4$ $\bar{k}=1$ $\underline{i}=1$ $\bar{i}=-2$
$u = \log L$ $u_0 = \log L_u$
$H = e^{k(u-u_0)}$, $\underline{H} = e^{\underline{k}(u-u_0)}$, $\bar{H} = e^{\bar{k}(u-u_0)}$

EE541−7N

double line element of *Richter* (1987) for the lighting technology with the luminance $L = F(L_P, M_D, S_T)$

$$F(L) = \int_{-\infty}^{L} (L/\Delta L)\, dL \quad \text{(relative lightness?)}$$
$$F(L) = iQ(H) \qquad H = e^{k(u-u_0)}$$
$$Q(H) = [\ln\{1+1/(1+\sqrt{2}H)\}]/\ln\sqrt{2} - 1$$

Taylor-derivations:

$$\Delta F(L) = \frac{dF}{dL}\Delta L = i\, \frac{dQ}{dH}\Delta H$$
$$= -i\sqrt{2}\Delta H/[\ln\sqrt{2}(1+\sqrt{2}H)(2+\sqrt{2}H)]$$

EE541−8N

TUB-test chart EE54; Achromatic thresholds, contrast *(L/ΔL)*, and lightness *L**
Line element, contrast, and lightness according to *Weber-Fechner, Stiles, and Vos&Walraven*