

Line-element examples for grey samples ($0,2 \leq x \leq 5$)

$F(x)$ is called the line-element function of $f(x)$.
The following relations are valid for $x=Y/Y_u=Y/18$:

$$\frac{d[F(x)]}{dx} = f(x) \quad [1]$$

$$F(x) = \int \frac{f'(x)}{f(x)} dx \quad [2]$$

Example for the normalized tristimulus value $x=Y/Y_u$:

$$\frac{d[\ln(1+b \cdot x)]}{dx} = \frac{ab}{1+b \cdot x} \quad [3]$$

$$a \ln(1+b \cdot x) = \int \frac{ab}{1+b \cdot x} dx \quad [4]$$

DEQ60-1N

Line-element examples for grey samples ($0,2 \leq x \leq 5$)

$F_u(x)$ is called the line-element function of $f_u(x)$.
Both functions are normalized to the surround value:

$$\frac{d[F_u(x)]}{dx} = f_u(x) \quad [1]$$

$$F_u(x) = \int \frac{f'_u(x)}{f_u(x)} dx = \int \frac{b}{1+b \cdot x} dx \quad [2]$$

Example for $L^*(x)$ & ΔY with $x=Y/Y_u$, $x_u=1$, $b=6,141$:

$$L^*_u(x) = \frac{L^*(x)}{L^*(x_u)} = \frac{\ln(1+b \cdot x)}{\ln(1+b)} \quad [3]$$

$$f_u(x) = \frac{\Delta Y}{\Delta Y_u} = \frac{1+b \cdot x}{1+b} \quad [4]$$

DEQ60-3N

Line-element equations according to CIE 230:2019

Colour-discrimination function $f(x) = \Delta Y = \Delta x \cdot Y_u$ [0]
 $\Delta Y = (A_1 + A_2 Y)/A_0$ $A_0=1,5$, $A_1=0,0170$, $A_2=0,0058$

$$f_u(x) = \frac{\Delta Y}{\Delta Y_u} = \frac{1+b \cdot x}{1+b} \quad b=A_2 Y_u/A_1 \quad x=Y/Y_u \quad [1]$$

$$F_u(x) = \int \frac{f'_u(x)}{f_u(x)} dx = \int \frac{b}{1+b \cdot x} dx \quad [2]$$

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DEQ60-5N

Line-element equations for thresholds and scaling

Colour-discrimination function $f(x) = \Delta Y = \Delta x \cdot Y_u$ [0]
 $\Delta Y = 1/[y(1+y)] = 1/y - 1/(1+y)$ $y=(1+\sqrt{2}) e^{k(u-u_0)}$

$$f_u(x) = \frac{\Delta Y}{\Delta Y_u} = \frac{1+x}{2} - \frac{2+x}{3} \quad x=Y/Y_u \quad [1]$$

$$F_u(x) = \int \frac{f'_u(x)}{f_u(x)} dx = \int \frac{1}{1+x} dx - \int \frac{1}{2+x} dx \quad [2]$$

Example for $L^*(x)$ & ΔY with $x=Y/Y_u$, $x_u=1$:

$$L^*_u(x) = \frac{L^*(x)}{L^*(x_u)} = \frac{\ln(1+x)}{\ln(2)} - \frac{\ln(1+y)}{\ln(1,5)} \quad [3]$$

$$f_u(x) = \frac{\Delta Y}{\Delta Y_u} = \frac{1+x}{2} - \frac{1+0,5x}{1,5} \quad [4]$$

see K. Richter (1985), Computer Graphic and Colorimetry, p. 113-127
<http://color.li.tu-berlin.de/BUA4BF.PDF>

DEQ60-7N

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$F_u(x)$ is called the line-element function of $f_u(x)$.
Both functions are normalized to the surround value:

$$\frac{d[F_u(x)]}{dx} = f_u(x) \quad [1]$$

$$F_u(x) = \int \frac{f'_u(x)}{f_u(x)} dx \quad [2]$$

Example for the normalized functions with $x_u=1$:

$$F_u(x) = \frac{F(x)}{F(x_u)} = \frac{\ln(1+b \cdot x)}{\ln(1+b)} \quad [3]$$

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DEQ60-2N

Line-element equations according to CIE 230:2019

Colour-threshold (t) function $f_t(x) = \Delta Y_t = \Delta x \cdot Y_u$ [0]
 $\Delta Y_t = (A_1 + A_2 Y)/A_0$ $A_0=1,5$, $A_1=0,0170$, $A_2=0,0058$

$$f_{tu}(x) = \frac{\Delta Y_t}{\Delta Y_{tu}} = \frac{1+b \cdot x}{1+b} \quad b=A_2 Y_u/A_1 \quad x=Y/Y_u \quad [1]$$

$$F_{tu}(x) = \int \frac{f'_{tu}(x)}{f_{tu}(x)} dx = \int \frac{b}{1+b \cdot x} dx \quad [2]$$

Example for $L^*(x)$ & ΔY_t with $x=Y/Y_u$, $x_u=1$, $b=6,141$:

$$L^*_{tu}(x) = \frac{L^*(x)}{L^*(x_u)} = \frac{\ln(1+b \cdot x)}{\ln(1+b)} \quad [3]$$

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DEQ60-4N

Line-element equations for thresholds and scaling

Colour-discrimination function $f(x) = \Delta Y = \Delta x \cdot Y_u$ [0]
 $\Delta Y = 1/[y(1+y)] = 1/y - 1/(1+y)$ $x=\sqrt{2} e^{k(u-u_0)}$

$$f_u(x) = \frac{\Delta Y}{\Delta Y_u} = \frac{1+b \cdot x}{1+b} - \frac{1+0,5b \cdot x}{1+0,5b} \quad b=1, \quad x=Y/Y_u \quad [1]$$

$$F_u(x) = \int \frac{f'_u(x)}{f_u(x)} dx = \int \frac{b}{1+b \cdot x} dx - \int \frac{0,5b}{1+0,5b \cdot x} dx \quad [2]$$

Example for $L^*(x)$ & ΔY with $x=Y/Y_u$, $x_u=1$, $b=1$:

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DEQ60-6N

Line-element equations for thresholds and scaling

Colour-discrimination function $f(y) = \Delta Y = \Delta y \cdot Y_u$ [0]
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$$f_u(y) = \frac{\Delta Y}{\Delta Y_u} = \frac{y}{2} - \frac{1+y}{3} \quad y=1+Y/Y_u, \quad dy=dx \quad [1]$$

$$F_u(y) = \int \frac{f'_u(y)}{f_u(y)} dy = \int \frac{1}{y} dy - \int \frac{1}{1+y} dy \quad [2]$$

Example for $L^*(y)$ & ΔY with $y=1+Y/Y_u$, $y_u=2$:

$$L^*_u(y) = \frac{L^*(y)}{L^*(y_u)} = \frac{\ln(1+y)}{\ln(2)} - \frac{\ln(1+y)}{\ln(3)} \quad [3]$$

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DEQ60-8N

Line-element examples for grey samples ($0,2 \leq Y_r \leq 5$)

$F(Y_r)$ is called the line-element function of $f(Y_r)$.
The following relations are valid for $Y_r=Y/Y_u=Y/18$:

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Example for the normalized tristimulus value $Y_r=Y/Y_u$:

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DEQ61-1N

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$F_u(Y_r)$ is called the line-element function of $f_u(Y_r)$.
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Example for $L^*(Y_r)$ & ΔY_r with $Y_{ru}=1$, $b=6,141$:

$$L^*_{u}(Y_r) = \frac{L^*(Y_r)}{L^*(Y_{ru})} = \frac{\ln(1+b \cdot Y_r)}{\ln(1+b)} \quad [3]$$

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DEQ61-3N

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DEQ61-5N

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DEQ61-7N

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DEQ61-4N

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DEQ61-6N

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DEQ61-8N