Weber-Fechner law in CIE 230:2019 for threshold colour differences of surface colours

The *Weber-Fechner* law describes the lightness L^*_r as ***logarithmic*** function of L_r.
The *Stevens* law describes the lightness L^*_{CIELAB} as ***potential*** function of $L_r=Y/5$.
IEC 61966-2-1 uses a similar potential function $L^*_{IEC} = m\,L_r^{1/2,4}$.
The *Weber-Fechner* law is equivalent to the equation: $\Delta L_r = c\,L_r$ [1]
Integration leads to the logarithmic equation: $L^*_r = k\log(L_r)$. [2]
Derivation for $\Delta L^*_r=1$ leads to the linear equation: $L_r/\Delta L_r = k = 57$. [3]
For colours in offices the **standard contrast range** is **25:1**=90:3,6.

Table 1: CIE tristimulus value *Y*, luminance *L*, and lightnesses *L**

Colour (matte)	Tritimulus value	office luminance	relative luminance	CIE lightness	relative lightness
***(contrast)* (25:1=90:3,6)**	***Y***	***L* [cd/m²]**	**L_r $=L/L_u$**	**L^*_{CIELAB} $\sim m\,L_r^{1/2,4}$**	**L^*_r $=k\log(L_r)$**
White W (paper)	90 =18*5	142 =28,2*5	5	94 =50+44	40 $=k\log(5)$
Grey Z (paper)	18	28,2	1	50	0 $=k\log(1)$
Black N (paper)	3,6 =18/5	5,6 28,2/5	0,2	18 50−32	−40 $=k\log(0,2)$

For the lightness range between $L^*_r=-40$ and 40 the constant is: $k=40/\log(5)=57$

DEH00−1N

Weber-Fechner law in CIE 230:2019 for threshold colour differences of surface colours and two ranges 0,2 <= L_r <=1 and 1 <= L_r <=5

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IEC 61966-2-1 uses a similar potential function $L^*_{IEC} = m\,L_r^{1/2,4}$.
The *Weber-Fechner* law is equivalent to the linear equation: $\Delta L_r = c_i\,L_r$ $(i=0,1)$ [1]
Integration leads to the logarithmic equation: $L^*_r = k_i\log(L_r)$ $(i=0,1)$ [2]
Derivation leads for $\Delta L^*_r=1$ to the linear equation: $L_r/\Delta L_r = k_i$ ($k_0=46$, $k_1=63$) [3]
For colours in offices the **standard contrast range** is **25:1**=90:3,6.

Table 1: CIE tristimulus value *Y*, luminance *L*, and lightnesses *L**

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Grey Z (paper)	18	28,2	1	50	0 $=k_0\log(1)$
Black N (paper)	3,6 =18/5	5,6 28,2/5	0,2	18 50−32	−32 $=k_0\log(0,2)$

For the two lightness ranges it is $k_0=-32/\log(0,2)=46$ and $k_1=44/\log(5)=63$.

DEH00−2N

line element of *Stiles*
(1946) with „color values” L_P, M_D, S_T

three separate color signal functions

$$F(L_P)=i\ln(1+9L_P)$$
$$F(M_D)=j\ln(1+9M_D)$$
$$F(S_T)=k\ln(1+9S_T)$$

***Taylor*-derivations:**

$$\Delta F(L_P,M_D,S_T)=\frac{dF}{dL_P}\Delta L_P+\frac{dF}{dM_D}\Delta M_D+\frac{dF}{dS_T}\Delta S_T$$
$$=\frac{9i}{1+9L_P}\Delta L_P+\frac{9j}{1+9M_D}\Delta M_D+\frac{9k}{1+9S_T}\Delta S_T$$

DEH01−1N

line element of *Vos&Walraven*
(1972) with „color values” L_P, M_D, S_T

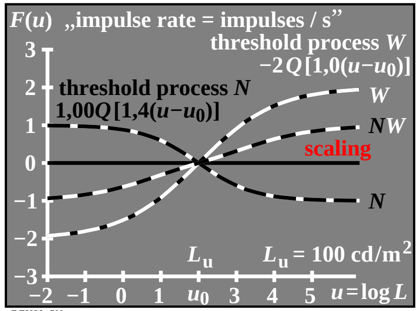
three separate color signal functions

$$F(L_P)=-2i\sqrt{L_P}$$
$$F(M_D)=-2j\sqrt{M_D}$$
$$F(S_T)=-2k\sqrt{S_T}$$

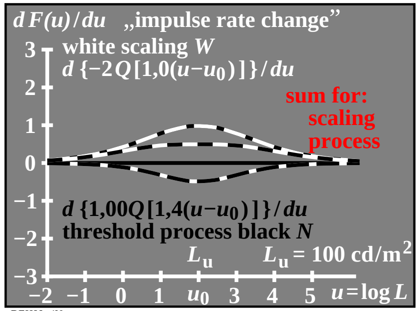
***Taylor*-derivations:**

$$\Delta F(L_P,M_D,S_T)=\frac{dF}{dL_P}\Delta L_P+\frac{dF}{dM_D}\Delta M_D+\frac{dF}{dS_T}\Delta S_T$$
$$\Delta F(L_P,M_D,S_T)=\frac{i}{\sqrt{L_P}}\Delta L_P+\frac{j}{\sqrt{M_D}}\Delta M_D+\frac{k}{\sqrt{S_T}}\Delta S_T$$

DEH01−2N



DEH00−3N



DEH00−4N

functions $q[k(u-u_0)]$
„achromatic signal”-description

with $u=\log L$ (L = luminance)
$u_0=\log L_u$ (L_u = surround luminance)

$q[k(u-u_0)]=1+1/[1+\sqrt{2}\,e^{k(u-u_0)}]$

function values:

$q[k(u-u_0)\to+\infty] = 1$
$q[k(u-u_0)=0] = \sqrt{2}$
$q[k(u-u_0)\to-\infty] = 2$

DEH01−3N

„achromatic signal”-description
functions $Q_{1m}[k(u-u_0)]$

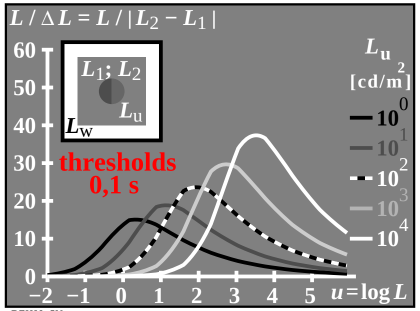
with $u=\log L$ (L = luminance)
$u_0=\log L_u$ (L_u = surround luminance)

$Q_{1m}[k(u-u_0)]=\frac{l}{\ln\sqrt{2}}\ln q[k(u-u_0)]-m$

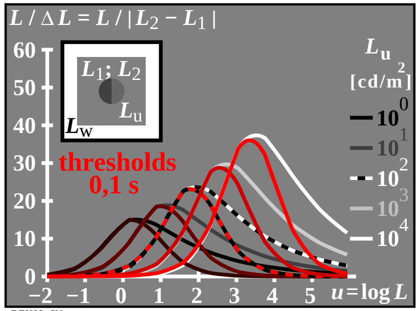
function values with l = m = 1:

$Q[k(u-u_0)\to+\infty] = -1$
$Q[k(u-u_0)=0] = 0$
$Q[k(u-u_0)\to-\infty] = 1$

DEH01−4N



DEH00−5N



DEH00−6N

„achromatic signal” discrimination
as function of relative light density
$h=\ln H=k(u-u_0)$, ln = natural log.

$$Q'=\frac{d}{dH}[\ln\{1+1/(1+\sqrt{2}H)\}]/\ln\sqrt{2}$$
$$=-\sqrt{2}/[\ln\sqrt{2}(1+\sqrt{2}H)(2+\sqrt{2}H)]$$

function values:

$Q'[k(u-u_0)\to+\infty] = 0$
$Q'[k(u-u_0)=0] = -0,5$
$Q'[k(u-u_0)\to-\infty] = 0$

DEH01−5N

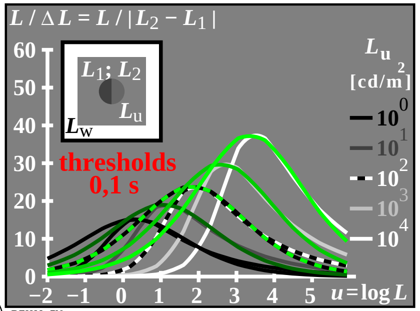
luminance discrimination
possibility $L/\Delta L$ as function of H

with: $L=10^u$ $H=e^h=10^{\log e\,k(u-u_0)}$
$dL/du=\ln 10\,L$ $dH/du=k\,H$

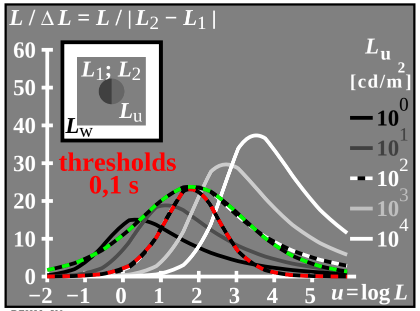
it follows: $L/\Delta L=[k\,H/(dH\ln 10)]$
$$\frac{L}{dL}=\text{const}\,H/[(1+\sqrt{2}H)(2+\sqrt{2}H)]$$

$Q'[k(u-u_0)\to+\infty] = 0$
$Q'[k(u-u_0)=0] =$ maximum
$Q'[k(u-u_0)\to-\infty] = 0$

DEH01−6N



DEH00−7N



DEH00−8N

double line element of *Richter*
(1987) for the lighting technology with
the luminance $L=f(L_P, M_D, S_T)$

$F(L)=\int_{-\infty}^{L}(L/\Delta L)\,dL$ (relative L, M, S?)

$$F(L)=i\,Q(H)=\begin{cases}\underline{i}\,Q(\underline{H}) & (u<u_0)\\ \bar{i}\,Q(\bar{H}) & (u\geq u_0)\end{cases}$$

with: $\underline{k}=1,4$ $\bar{k}=1$ $\underline{i}=1$ $\bar{i}=-2$
$u=\log L$ $u_0=\log L_u$
$H=e^{k(u-u_0)}$, $\underline{H}=e^{\underline{k}(u-u_0)}$, $\bar{H}=e^{\bar{k}(u-u_0)}$

DEH01−7N

double line element of *Richter*
(1987) for the lighting technology with
the luminance $L=F(L_P, M_D, S_T)$

$F(L)=\int_{-\infty}^{L}(L/\Delta L)\,dL$ (relative L, M, S?)
$F(L)=i\,Q(H)$ $H=e^{k(u-u_0)}$
$Q(H)=[\ln\{1+1/(1+\sqrt{2}H)\}]/\ln\sqrt{2}-1$

Taylor-derivations:

$$\Delta F(L)=\frac{dF}{dL}\Delta L=i\,\frac{dQ}{dH}\Delta H$$
$$=-i\sqrt{2}\,\Delta H/[\ln\sqrt{2}(1+\sqrt{2}H)(2+\sqrt{2}H)]$$

DEH01−8N

TUB-test chart DEH0; Thresholds, contrast ($L/\Delta L$), and lightness L^* input: *rgb/cmy0/000k/n*
Line element, contrast, and lightness according to *Weber-Fechner, Stiles, and Vos&Walraven*

see similar files: http://farbe.li.tu-berlin.de/DEH0/DEH0.HTM
technical information: http://farbe.li.tu-berlin.de or http://color.li.tu-berlin.de

TUB registration: 20210901-DEH0/DEH0L0NA.TXT /.PS
application for evaluation and measurement of display or print output
TUB material: code=rha4ta