

Weber-Fechner law in CIE 230:2019 for threshold colour differences of surface colours

The Weber-Fechner law describes the lightness L^*_r as *logarithmic* function of L_r .
 The Stevens law describes the lightness L^*_{CIELAB} as *potential* function of $L_r = Y/5$.
 IEC 61966-2-1 uses a similar potential function $L^*_{\text{IEC}} = m L_r^{1/2,4}$.

The Weber-Fechner law is equivalent to the equation: $\Delta L_r = c L_r$ [1]

Integration leads to the logarithmic equation: $L^*_r = k \log(L_r)$. [2]

Derivation for $\Delta L^*_r = 1$ leads to the linear equation: $L_r / \Delta L_r = k = 57$. [3]

For colours in offices the standard contrast range is 25:1=90:3,6.

Table 1: CIE tristimulus value Y , luminance L , and lightnesses L^*

Colour (matte)	Tritimulus value	office luminance	relative luminance	CIE lightness	relative lightness
(contrast) (25:1=90:3,6)	Y	L [cd/m ²] $=L/L_u$	L_r $=L/L_u$	L^*_{CIELAB} $\sim m L_r^{1/2,4}$	L^*_r $=k \log(L_r)$
White W (paper)	90 $=18*5$	142 $=28,2*5$	5	94 $=50+44$	40 $=k \log(5)$
Grey Z (paper)	18	28,2	1	50	0 $=k \log(1)$
Black N (paper)	3,6 $=18/5$	5,6 $=28,2/5$	0,2	18 $=50-32$	-40 $=k \log(0,2)$

For the lightness range between $L^*_r = -40$ and 40 the constant is: $k = 40/\log(5) = 57$