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Transformation between the *Iudd* tristimulus and opponent values
 Data from K. Richter, PhD thesis, University of Basel (Switzerland), 1969, page 8.
 For the antagonistic spectral elementary colours
 $\lambda_{41} = 475 \text{ nm}$, $\lambda_2 = 502 \text{ nm}$, $\lambda_3 = 574 \text{ nm}$, $\lambda_4 = 494 \text{ nm}$
 the coordinates x_i ($i = 1$ to 3) are used instead of modern coordinates $\bar{x}$, $\bar{a}$, $\bar{b}$.
 Linear model equations between spectral colour values in both directions:

$$\begin{pmatrix} \bar{x}(\lambda) \\ \bar{a}(\lambda) \\ \bar{b}(\lambda) \end{pmatrix} = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix} \begin{pmatrix} x(\lambda) \\ y(\lambda) \\ z(\lambda) \end{pmatrix} = \begin{pmatrix} 0.0000 & 1.0000 & 0.0000 \\ 2.9797 & -2.6652 & -0.0960 \\ -0.4139 & 1.4571 & -2.4046 \end{pmatrix} \begin{pmatrix} x(\lambda) \\ y(\lambda) \\ z(\lambda) \end{pmatrix} \quad (1)$$

$$\begin{pmatrix} x(\lambda) \\ y(\lambda) \\ z(\lambda) \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} \bar{x}(\lambda) \\ \bar{a}(\lambda) \\ \bar{b}(\lambda) \end{pmatrix} = \begin{pmatrix} 0.9093 & 0.3338 & -0.1133 \\ 1.0000 & 0.0000 & 0.0000 \\ 0.4494 & -0.0574 & -0.4136 \end{pmatrix} \begin{pmatrix} \bar{x}(\lambda) \\ \bar{a}(\lambda) \\ \bar{b}(\lambda) \end{pmatrix} \quad (2)$$

The tristimulus values X , Y , Z and x , Y , Z need the same transformations.

The unnormalized purity data a_i and b_i are defined in LabMUN 1969 as follows:
 $a_i = X_i/X - X_0/X_0$, $b_i = Y_i/Y - Y_0/Y_0$, $x_i = 1 - x_0 - y_0$
 The unnormalized purity data a_i and b_i are defined in LabMUN 1969 as follows:
 $a_i = (b_{21} - b_{23})x_i + (b_{22} - b_{23})y_i + b_{23}z_i$
 $= (3.0757x - 2.5702y - 0.0960z)$
 $b_i = (b_{31} - b_{33})x_i + (b_{32} - b_{33})y_i + b_{33}z_i$
 $= (1.9906x + 3.8617y - 2.4046z)$

Transformation between the Judd tristimulus and opponent values

See K. Richter, PhD thesis, University of Basel (Switzerland), 1969, page 81

For the antagonistic spectral elementary colours

$\lambda_{41} = 475$ nm, $\lambda_2 = 502$ nm, $\lambda_3 = 574$ nm, $\lambda_4 = 494$ nm

modern coordinates l, a, b are used instead of $\bar{x}_i$ ($i=1$ to 3).

Linear model equations between spectral colour values in both directions:

$$\begin{pmatrix} \bar{l}(x) \\ \bar{a}(x) \\ \bar{b}(x) \end{pmatrix} = \begin{pmatrix} b_{21} & b_{12} & b_{13} \\ b_{31} & b_{22} & b_{23} \\ b_{41} & b_{32} & b_{33} \end{pmatrix} \begin{pmatrix} \bar{x}(x) \\ \bar{y}(x) \\ \bar{z}(x) \end{pmatrix} = \begin{pmatrix} 0.0000 & 1.0000 & 0.0000 \\ 2.9797 & -2.6652 & -0.0960 \\ -0.4139 & 1.4571 & -2.4046 \end{pmatrix} \begin{pmatrix} \bar{x}(x) \\ \bar{y}(x) \\ \bar{z}(x) \end{pmatrix} \quad (1)$$

$$\begin{pmatrix} \bar{x}(x) \\ \bar{y}(x) \\ \bar{z}(x) \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} \bar{l}(x) \\ \bar{a}(x) \\ \bar{b}(x) \end{pmatrix} = \begin{pmatrix} 0.9093 & 0.3338 & -0.1133 \\ 1.0000 & 0.0000 & 0.0000 \\ 0.4494 & -0.0574 & -0.4136 \end{pmatrix} \begin{pmatrix} \bar{l}(x) \\ \bar{a}(x) \\ \bar{b}(x) \end{pmatrix} \quad (2)$$

The tristimulus values l, a, b and X, Y, Z see the same transformations.

The unnormalized purity data a_i and b_i are defined in LabMun 1969 as follows:

$$a_i = X/Y - x/y \quad (3) \quad b_i = -2Y/Z - y/z \quad (4) \quad z = 1 - x - y \quad (5)$$

The purity data a_i and b_i are defined in LabMun 1969 as follows:

$$a_i = (b_{21} - b_{22} + k(b_{22} - b_{23}) + b_{23})/y$$

$$= (3.0757 \cdot x - 2.5702 \cdot y - 0.0960)/y \quad (6)$$

$$b_i = (b_{31} - b_{32} + k(b_{32} - b_{33}) + b_{33})/y$$

$$= (1.9906 \cdot x + 3.6171 \cdot y - 2.4046)/y \quad (7)$$

Table presented by the *Judd tristimulus and opponent values*

Data from K. Richter, PhD thesis, University of Basel (Switzerland), 1969, page 3

elementary colour	dominant wavelength	Judd spectral tristimulus values		chromatic value	
		$\bar{x}(\lambda)$	$\bar{y}(\lambda)$	RG $\bar{x}(\lambda)$	YB $\bar{y}(\lambda)$
blue	$\lambda = 475 \text{ nm}$	0.8267	0.9339	0.0017	0.0000
green	$\lambda = 502 \text{ nm}$	0.0107	0.0038	0.0005	-0.0000, 0.0000
yellow	$\lambda = 574 \text{ nm}$	0.1304	0.1124	0.9281	0.0000, 0.0000
red	$\lambda = 494\text{--}496 \text{ nm}$	0.0028	0.3701	0.2238	-0.0000

There are six equations to calculate the six constants: b_1 to b_3

$$\begin{aligned} \bar{x}(\lambda_n) &= b_1(\bar{x}(\lambda_1) + b_2(\bar{x}(\lambda_2) + b_3(\bar{x}(\lambda_3) + \bar{x}(\lambda_4) + \bar{x}(\lambda_5) + \bar{x}(\lambda_6)) \\ \bar{y}(\lambda_n) &= b_1(\bar{y}(\lambda_1) + b_2(\bar{y}(\lambda_2) + b_3(\bar{y}(\lambda_3) + \bar{y}(\lambda_4) + \bar{y}(\lambda_5) + \bar{y}(\lambda_6)) \\ \bar{x}(\lambda_n) &= b_1(\bar{x}(\lambda_1) + b_2(\bar{x}(\lambda_2) + b_3(\bar{x}(\lambda_3) + \bar{x}(\lambda_4) + \bar{x}(\lambda_5) + \bar{x}(\lambda_6)) \\ \bar{y}(\lambda_n) &= b_1(\bar{y}(\lambda_1) + b_2(\bar{y}(\lambda_2) + b_3(\bar{y}(\lambda_3) + \bar{y}(\lambda_4) + \bar{y}(\lambda_5) + \bar{y}(\lambda_6)) \\ \bar{x}(\lambda_n) &= b_1(\bar{x}(\lambda_1) + b_2(\bar{x}(\lambda_2) + b_3(\bar{x}(\lambda_3) + \bar{x}(\lambda_4) + \bar{x}(\lambda_5) + \bar{x}(\lambda_6)) \\ \bar{y}(\lambda_n) &= b_1(\bar{y}(\lambda_1) + b_2(\bar{y}(\lambda_2) + b_3(\bar{y}(\lambda_3) + \bar{y}(\lambda_4) + \bar{y}(\lambda_5) + \bar{y}(\lambda_6)) \end{aligned}$$

Together with the use of the standard equation: $\bar{x}(\lambda) = \bar{y}(\lambda)$

the equations between spectral opponent tristimulus colour values are:

$$\begin{aligned} \bar{x}(\lambda) &= \begin{pmatrix} b_1 & b_2 & b_3 \end{pmatrix} \begin{pmatrix} \bar{x}(\lambda_1) \\ \bar{x}(\lambda_2) \\ \bar{x}(\lambda_3) \end{pmatrix} = \begin{pmatrix} 0.0000 & 1.0000 & 0.0000 \end{pmatrix} \begin{pmatrix} \bar{x}(\lambda) \\ \bar{y}(\lambda) \\ \bar{z}(\lambda) \end{pmatrix} \\ \bar{y}(\lambda) &= \begin{pmatrix} b_1 & b_2 & b_3 \end{pmatrix} \begin{pmatrix} \bar{y}(\lambda_1) \\ \bar{y}(\lambda_2) \\ \bar{y}(\lambda_3) \end{pmatrix} = \begin{pmatrix} 2.9797 & -2.6652 & -0.0960 \end{pmatrix} \begin{pmatrix} \bar{x}(\lambda) \\ \bar{y}(\lambda) \\ \bar{z}(\lambda) \end{pmatrix} \\ \bar{z}(\lambda) &= \begin{pmatrix} b_1 & b_2 & b_3 \end{pmatrix} \begin{pmatrix} \bar{z}(\lambda_1) \\ \bar{z}(\lambda_2) \\ \bar{z}(\lambda_3) \end{pmatrix} = \begin{pmatrix} -0.4139 & 1.4571 & -2.4046 \end{pmatrix} \begin{pmatrix} \bar{x}(\lambda) \\ \bar{y}(\lambda) \\ \bar{z}(\lambda) \end{pmatrix} \end{aligned}$$

Remark: The weighting ratio in the RG and YB direction is 2.8:1 or 1:0.3571.

Data between the *Judd* tristimulus and opponent values
 Take see K. Richter, PhD thesis, University of Basel (Switzerland), 1969, page 3

elementary colour	dominant wavelength	<i>Judd</i> spectral tristimulus values $\bar{x}(\lambda)$ $\bar{y}(\lambda)$ $\bar{z}(\lambda)$	chromatic value RG YB $\alpha(\lambda)$ $\beta(\lambda)$
blue	$\lambda_{\text{blue}}=475 \text{ nm}$	0.8267 0.9339 0.0017	0,0000 -
green	$\lambda_{\text{green}}=502 \text{ nm}$	0.0107 0.0038 0.0005	-1,0000 0,0000
yellow	$\lambda_{\text{yellow}}=574 \text{ nm}$	0.1304 0.1124 0.9281	0,0000 1,0000
red	$\lambda_{\text{red}}=494\text{E}.\text{nm}$	0.0028 0.3701 0.2238	-1,0000 -

There are six equations to calculate the six constants: b_1 to b_3

$$\begin{aligned}\alpha(\lambda_i) &= b_1(\bar{x}_i(\lambda_i) + b_2(\bar{x}_i(\lambda_j)) + b_3(\bar{x}_i(\lambda_k)) - \bar{x}_i(\lambda_i)) & \delta(\lambda_i) &= b_1(\bar{x}_i(\lambda_i) + b_2(\bar{x}_i(\lambda_j) + b_3(\bar{x}_i(\lambda_k))) \\ \alpha(\lambda_j) &= b_1(\bar{x}_j(\lambda_i) + b_2(\bar{x}_j(\lambda_j)) + b_3(\bar{x}_j(\lambda_k)) - 1 & \gamma(\lambda_i) &= b_1(\bar{x}_i(\lambda_i) + b_2(\bar{x}_i(\lambda_j) + b_3(\bar{x}_i(\lambda_k))) \\ \alpha(\lambda_k) &= b_1(\bar{x}_k(\lambda_i) + b_2(\bar{x}_k(\lambda_j)) + b_3(\bar{x}_k(\lambda_k)) - 1 & \gamma(\lambda_j) &= b_1(\bar{x}_j(\lambda_i) + b_2(\bar{x}_j(\lambda_j) + b_3(\bar{x}_j(\lambda_k))) \\ \beta(\lambda_i) &= b_1(\bar{y}_i(\lambda_i) + b_2(\bar{y}_i(\lambda_j)) + b_3(\bar{y}_i(\lambda_k)) - \bar{y}_i(\lambda_i) & \gamma(\lambda_k) &= b_1(\bar{x}_k(\lambda_i) + b_2(\bar{x}_k(\lambda_j) + b_3(\bar{x}_k(\lambda_k))) \\ \beta(\lambda_j) &= b_1(\bar{y}_j(\lambda_i) + b_2(\bar{y}_j(\lambda_j)) + b_3(\bar{y}_j(\lambda_k)) - 1 & \end{aligned}$$

Together with the use of the standard equation: $l(\lambda)=\bar{y}(\lambda)$

The equations between spectral opponent and chromatic values are:

$$\begin{aligned}\frac{\alpha(\lambda)}{l(\lambda)} &= \frac{b_1}{b_{12}} \cdot \frac{b_2}{b_{12}} \cdot \frac{b_3}{b_{12}} \cdot \left(\frac{\bar{x}(\lambda)}{0.0000} - \frac{1.0000}{0.0000} \right) & \frac{\delta(\lambda)}{l(\lambda)} &= \frac{b_1}{b_{12}} \cdot \frac{b_2}{b_{12}} \cdot \frac{b_3}{b_{12}} \cdot \left(\frac{\bar{x}(\lambda)}{0.0000} - \frac{1.0000}{0.0000} \right) \\ \frac{\beta(\lambda)}{l(\lambda)} &= \frac{b_1}{b_{12}} \cdot \frac{b_2}{b_{12}} \cdot \frac{b_3}{b_{12}} \cdot \left(\frac{\bar{y}(\lambda)}{2.9797} - \frac{2.6562}{-0.0960} - \frac{0.0960}{-0.0960} \right) & \frac{\gamma(\lambda)}{l(\lambda)} &= \frac{b_1}{b_{12}} \cdot \frac{b_2}{b_{12}} \cdot \frac{b_3}{b_{12}} \cdot \left(\frac{\bar{x}(\lambda)}{-0.4139} - \frac{1.4571}{-2.4046} - \frac{2.4046}{-2.4046} \right)\end{aligned}$$

Remark: The weighting ratio in the RG and YB direction is 2.8:1 or 1:0.3571.

Transformation between the *Judd* tristimulus and opponent values
 Data from K. Richter, PhD thesis, University of Basel (Switzerland), 1969, page 8.
 For the antagonistic spectral elementary colours
 $\lambda_{40} = 475$ nm, $\lambda_5 = 502$ nm, $\lambda_6 = 574$ nm, $\lambda_7 = 494$ nm
 the coordinates x_i ($i = 1$ to 3) are used instead of modern coordinates $\bar{x}$, $\bar{a}$, $\bar{b}$.
 Linear model equations between spectral colour values in both directions:

$$\begin{pmatrix} \bar{x}(\lambda) \\ \bar{a}(\lambda) \\ \bar{b}(\lambda) \end{pmatrix} = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix} \begin{pmatrix} x(\lambda) \\ y(\lambda) \\ z(\lambda) \end{pmatrix} = \begin{pmatrix} 0.0000 & 1.0000 & 0.0000 \\ 2.9797 & -2.6662 & -0.0960 \\ -0.4139 & 1.4571 & -2.4046 \end{pmatrix} \begin{pmatrix} x(\lambda) \\ y(\lambda) \\ z(\lambda) \end{pmatrix} \quad (1)$$

$$\begin{pmatrix} x(\lambda) \\ y(\lambda) \\ z(\lambda) \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} \bar{x}(\lambda) \\ \bar{a}(\lambda) \\ \bar{b}(\lambda) \end{pmatrix} = \begin{pmatrix} 0.0000 & 0.0000 & 0.0000 \\ 0.4494 & -0.0574 & -0.4136 \\ 0.9906 & 3.8617 & -2.4046 \end{pmatrix} \begin{pmatrix} \bar{x}(\lambda) \\ \bar{a}(\lambda) \\ \bar{b}(\lambda) \end{pmatrix} \quad (2)$$

The tristimulus values x , y , z , X , Y , Z need the same transformations.
 The normalized primary x_i ($i = 1, 2, 3$) and X are defined in LabMun 1969 as follows:
 $a_i = n_i \rho_i / (x_i + n_i \rho_i)$, $x_i = 2.8 \cdot 10^3 \cdot \bar{x}$, $b_i = b_i(x_i, y_i, z_i)$, $(i = 1, 2, 3)$, $x = x(x, y, z)$, (3)
 The normalized primary data a_i and b_i are defined in LabMun 1969 as follows:
 $a_i = n_i \cdot (b_i - b_{i3})x + (b_{i2} - b_{i3})y + b_{i3}z$
 $= 2.8(3.0757x - 2.5702y - 0.0960z)$
 $= (8.6119x - 7.1965y - 0.2688z)$ (4)
 $b_i = (b_{i1} - b_{i3})x + (b_{i2} - b_{i3})y + b_{i3}z$
 $= (1.9906x + 3.8617y - 2.4046z)$ (5)

Transformation between the *Judd* tristimulus and opponent values
 Data from K. Richter, PhD thesis, University of Basel (Switzerland), 1969, page 81

For the antagonistic spectral elementary colours
 $\lambda_{40} = 475 \text{ nm}$, $\lambda_C = 502 \text{ nm}$, $\lambda_V = 574 \text{ nm}$, $\lambda_R = 494 \text{ nm}$
 modern coordinates l , a , b are used instead of $\bar{x}_1$ ($l = 1$ to 3).

Linear model equations between spectral colour values in both directions:

$$\begin{pmatrix} l \\ a \\ b \end{pmatrix} = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix} \begin{pmatrix} \bar{x}(l) \\ \bar{x}(a) \\ \bar{x}(b) \end{pmatrix} = \begin{pmatrix} 0.0000 & 1.0000 & 0.0000 \\ 2.9797 & -2.6662 & -0.0960 \\ -0.4159 & -1.4371 & -2.4046 \end{pmatrix} \begin{pmatrix} \bar{x}(l) \\ \bar{x}(a) \\ \bar{x}(b) \end{pmatrix} \quad (1)$$

$$\begin{pmatrix} \bar{x}(l) \\ \bar{x}(a) \\ \bar{x}(b) \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} l \\ a \\ b \end{pmatrix} = \begin{pmatrix} 0.9093 & 0.3338 & -0.0133 \\ 1.0000 & 0.0000 & 0.0000 \\ 0.4494 & -0.0574 & -0.4136 \end{pmatrix} \begin{pmatrix} l \\ a \\ b \end{pmatrix} \quad (2)$$

The tristimulus values A , B , and X , Y , Z need the same transformations.

The normalized purple data a_1 and b_1 are defined in LabMunich 1969 as follows:
 $a_1 = n_1 x - n_2 Y / n_1 X - n_2 Y$, $n_1 = 2.8$ (3) $b_1 = b_2 - b_3 X / Y - Y$, $z = 1 - x - y$ (5)

The normalized purple data a_2 and b_2 are defined in LabMunich 1969 as follows:
 $a_2 = n_1 (b_1 - b_2) x + (b_2 - b_3) y + b_3 y$,
 $= 2.8 (3.0757 x - 2.5702 y - 0.0960 y)$
 $= (8.6119 x - 7.1965 y - 0.2688 y)$ (6)

$$b_2 = (1 - b_3) x + b_3 x + (b_2 - b_3) y + b_3 y$$

$$= (1.9906 x + 3.8617 y - 2.4046 y) \quad (7)$$

Figure 1 is a contour plot of the Munsell colour system, specifically showing Chroma-2 colour. The horizontal axis is labeled a_2 and ranges from -1 to 4. The vertical axis is labeled b_2 and ranges from -1 to 3. The plot features several concentric ellipses representing constant Value (1, 2, 5, 8, 9) and constant Chroma (2, 8, 9). A specific point is marked with a star and labeled 'C6'.

Equations for the axes are provided:

$$a_2 = a_{2s} - [(3,0757x - 2,5702y - 0,0960)$$

$$b_2 = b_{2s} - [(1,9906x + 3,8617y - 2,4046)$$

Value 1

Munsell colour system

Chroma-2 colour

Value 1, 2, 5, 8,

$a_{2s} = 2,8, b_{2s} = 1,0$

$\lambda_{B,G,Y,R} = 475, 503, 574, 494, \text{ nm}$

Transformation between the *Judd* purity and chromaticity
 data from K. Richter, PhD thesis, University of Basel (Switzerland), 1969, page 8)
 For the antagonistic spectral elementary colours
 $\lambda_B = 475 \text{ nm}$, $\lambda_C = 502 \text{ nm}$, $\lambda_Y = 574 \text{ nm}$, $\lambda_R = 494 \text{ nm}$
 the coordinates $\bar{x}_i$ ($i=1$ to 3) are used instead of modern coordinates $\bar{l}$, $\bar{a}$, $\bar{b}$.
 Relation between opponent and spectral colour values:

$$\begin{pmatrix} \bar{x}(X) \\ \bar{x}(Y) \\ \bar{x}(Z) \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} \bar{x}(X_0) \\ \bar{x}(Y_0) \\ \bar{x}(Z_0) \end{pmatrix} = \begin{pmatrix} 0.9093 & 0.3338 & -0.0133 \\ 1.0000 & 0.0000 & 0.0000 \\ 0.4494 & -0.4494 & -0.4136 \end{pmatrix} \begin{pmatrix} \bar{x}(X_0) \\ \bar{x}(Y_0) \\ \bar{x}(Z_0) \end{pmatrix} \quad (2)$$

 The tristimulus values X_0 , Y_0 , and Z_0 of X , Y , Z need the same transformations.
 The unnormalized purity data a_i and b_i are defined in LabMun 1969 as follows
 $a_i = X_i/X_0 - X_0/X_0$ (3) $b_i = Y_i/Y_0 - X_i/X_0$ (4) $x_1 = 1 - x_2 - x_3$ (5)
 Use of equations (2) to (5) leads to:

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} X_0 \\ Y_0 \\ Z_0 \end{pmatrix} \quad (6) \quad \text{and with } x = X/(X+Y+Z) \text{ and } y = Y/(X+Y+Z) \quad (7)$$

$$\begin{aligned} x &= [a_{11} \cdot a_{22} \cdot a_{33} \cdot a_{33}b_1] / [a_{11} \cdot a_{22} \cdot a_{33} \cdot (a_{11} + a_{22} + a_{33})b_1 + (a_{13} \cdot a_{22} + a_{23} \cdot a_{33})b_2] \\ &= [a_{11} \cdot a_{22} \cdot a_{33} \cdot a_{33}b_1] / [a_{11} \cdot a_{22} \cdot a_{33} \cdot (a_{11} + a_{22} + a_{33})b_1 + (a_{13} \cdot a_{22} + a_{23} \cdot a_{33})b_2] \quad (8) \\ x &= (a_{11} + a_{22} + a_{33}) \cdot (a_{11} + a_{22} + a_{33})b_1 \\ &= (0.9093 + 0.3338a_{33} + 0.0133b_3) \cdot (2.3587 + 0.2764a_{33} - 0.4269b_3) \quad (9) \\ y &= (a_{11} + a_{22} + a_{33}) \cdot (a_{11} + a_{22} + a_{33})b_2 \\ &= (1.0000 + 0.0000a_{33} + 0.0000b_3) \cdot (2.3587 + 0.2764a_{33} - 0.4269b_3) \quad (10) \end{aligned}$$

Transformation between the *Judd* priority and chromaticity
 data from K. Richter, PhD thesis, University of Basel (Switzerland), 1969, page 83

For the antagonistic spectral elementary chromaticities
 $\lambda_{\text{blue}} = 475 \text{ nm}$, $\lambda_{\text{c}} = 502 \text{ nm}$, $\lambda_{\text{y}} = 574 \text{ nm}$, $\lambda_{\text{red}} = 494 \text{ nm}$
 modern coordinates l , a , b are used instead of x_1 ($1 \rightarrow 3$).

Relation between opponent and spectral colour values:

$$\begin{pmatrix} \bar{x}(l) \\ \bar{y}(l) \\ \bar{z}(l) \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} l(l) \\ a(l) \\ b(l) \end{pmatrix} = \begin{pmatrix} 0.9093 & 0.3338 & -0.0133 \\ 1.0000 & 0.0000 & 0.0000 \\ 0.0000 & -0.0136 & 0.4136 \end{pmatrix} \begin{pmatrix} l(l) \\ a(l) \\ b(l) \end{pmatrix} \quad (2)$$

The tristimulus values l , a , b , and X , Y , Z need the same normalization. The unnormalized priority data a_i and b_i are defined in LabMUN 1969 as follows:
 $a_i = X/Y - Z/Y$ (3) $b_i = -Z/Y - z/y$ (4) $z = 1 - x - y$ (5)

Use of equations (2) to (5) leads to:
 $\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{bmatrix} l \\ a \\ b \end{bmatrix}$ (6) and with $x = X/(X+Y+Z)$ and $y = Y/(X+Y+Z)$ (7)
 $y = [a_{11} + a_{21}x + a_{31}z, a_{12} + a_{22}x + a_{32}z, a_{13} + a_{23}x + a_{33}z]$ (8)
 $x = [a_{11} + a_{21}x + a_{31}z, a_{12} + a_{22}x + a_{32}z, a_{13} + a_{23}x + a_{33}z]$ (9)
 $y = [a_{11} + a_{21}x + a_{31}z, a_{12} + a_{22}x + a_{32}z, a_{13} + a_{23}x + a_{33}z]$ (10)
 $y = [1.0000, 0.0000, 0.0000, 2.73587, 0.27646, -0.42693]$ (11)

Relation between the radial purity, P_r , of Chroma 2 and tristimulus value Y. Data see K. Richter, PhD thesis, University of Basel (Switzerland), 1969, page 1

Y	Y	x_M	y_M	a_{LM}	b_{LM}	x_U	y_U	a_{LU}	b_{LU}	P_{rM}
1,000	1,210	0,315	0,296	1,060	-2,130	0,314	0,324	0,310	-1,623	2,29
1,000	19,770	0,314	0,323	0,340	-1,640	0,314	0,324	0,310	-1,623	0,78
9,000	78,660	0,314	0,230	-1,550	0,314	0,324	0,310	-1,623	0,50	

$\log P_r$ radial purity of Chroma 2 hue circles.

$x = (0.9093 + 0.1192a - 0.0133b) / (2.3587 + 0.0987a - 0.4269b)$
 $y = (1.000 + 0.000a + 0.000b) / (2.3587 + 0.0987a - 0.4269b)$
 $r = -0.341$
 $P_r(F, M_Y) = P_r(F, M_{Y_{10}}) Y(F)$
 $n = M_{Y_{10}} = 2.20$
 $n = -0.341$

Value Y	sample data	surround data	purity
Y	x_m y_m a_{m1} b_{m1}	x_u y_u a_{u1} b_{u1}	p_r
1,000	1,210 3,315 0.296 1,060	-2,130 0.314 0.324 0.310	-1,623 2.29
2,000	3,130 1,314 0.308 0.720	-1,900 0.314 0.324 0.310	-1,623 2.46
3,000	6,560 3,134 0.315 0.560	-1,780 0.314 0.324 0.310	-1,623 2.44
4,000	12,900 3,134 0.320 0.480	-1,690 0.314 0.324 0.310	-1,623 2.60
5,000	17,700 3,134 0.323 0.430	-1,640 0.314 0.324 0.310	-1,623 2.78
5,570	25,300 3,134 0.324 0.410	-1,620 0.314 0.324 0.310	-1,623 2.82
6,000	30,050 3,134 0.325 0.300	-1,610 0.314 0.324 0.310	-1,623 6.69
7,000	40,360 3,134 0.326 0.270	-1,590 0.314 0.324 0.310	-1,623 6.69
8,000	59,100 3,134 0.327 0.250	-1,570 0.314 0.324 0.310	-1,623 5.54
9,000	78,660 3,134 0.328 0.230	-1,550 0.314 0.324 0.310	-1,623 4.33

For experimental (Ex) surround (hold data), see *Newhall*, JOSA 30 (1940) p. 62.

$$x = (\alpha_1 \alpha_2 \alpha_3 \alpha_4 \alpha_5 \alpha_6) \cdot (y_1 y_2 y_3 y_4 y_5 y_6) \cdot (1.0903 \cdot 0.1192_{a_1} - 0.1336_{b_1}) \cdot (2.3567 \cdot 0.0987_{a_2} - 0.4269_{b_2})$$

$$y = (\alpha_1 \alpha_2 \alpha_3 \alpha_4 \alpha_5 \alpha_6) \cdot (y_1 y_2 y_3 y_4 y_5 y_6) \cdot (1.0000 \cdot 0.000_{a_1} + 0.000_{b_1}) \cdot (2.3567 \cdot 0.0987_{a_2} - 0.4269_{b_2})$$

Transformation between the *Judd* purity and chromaticity
 Data from K. Richter, PhD thesis, University of Basel (Switzerland), 1969, page 8:
 For the antagonistic spectral elementary colours
 $\lambda_B = 475$ nm, $\lambda_G = 502$ nm, $\lambda_Y = 574$ nm, $\lambda_R = 494$ nm
 the coordinates X_i ($i=1$ to 3) are used instead of modern coordinates $\bar{x}$, $\bar{y}$, $\bar{z}$.
 Relation between opponent and spectral colour values:

$$\begin{pmatrix} \bar{x}(X_i) \\ \bar{y}(X_i) \\ \bar{z}(X_i) \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} = \begin{pmatrix} 0.9093 & 0.3338 & -0.0133 \\ 0.0000 & 0.0000 & 0.0000 \\ 0.4402 & 0.0574 & -0.4136 \end{pmatrix} \begin{pmatrix} \bar{x}(X_i) \\ \bar{y}(X_i) \\ \bar{z}(X_i) \end{pmatrix} \quad (2)$$

 The tristimulus values X_1 , X_2 , X_3 and X , Y , Z need the same transformations.
 The normalized purity data d , b and b_r are defined in LabMun 1969 as follows:
 $a_r = n_d a_r = n_d X = X - X_0$, $b_r = n_d Y = Y - Y_0$, $b = n_d Z = Z - Z_0$ (3)
 $b_r = b - X_0 Y = X_0 Y - X_0 Y$ (4) $X_0 = 1 - X - Y$ (5)
 Use of equations (2) to (5) leads to:

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} & b_{13} & b_{14} \\ a_{21} & a_{22} & a_{23} & b_{23} & b_{24} \\ a_{31} & a_{32} & a_{33} & b_{33} & b_{34} \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \\ X_3 \\ b \\ b_r \end{pmatrix} \quad (6)$$

 with $X = X(X+Y+Z)$ and $Y = Y(X+Y+Z)$ (7)

$$X = [a_{11} \cdot a_{12} \cdot a_{13} \cdot b_{13} \cdot b_{14}] \cdot [d_{11} \cdot d_{12} \cdot d_{13} \cdot d_{14} \cdot d_{15} \cdot d_{16} \cdot d_{17} \cdot d_{18} \cdot d_{19} \cdot d_{20} \cdot d_{21} \cdot d_{22} \cdot d_{23} \cdot d_{24} \cdot d_{25} \cdot d_{26} \cdot d_{27} \cdot d_{28} \cdot d_{29} \cdot d_{30} \cdot d_{31} \cdot d_{32} \cdot d_{33} \cdot d_{34} \cdot d_{35} \cdot d_{36} \cdot d_{37} \cdot d_{38} \cdot d_{39} \cdot d_{40} \cdot d_{41} \cdot d_{42} \cdot d_{43} \cdot d_{44} \cdot d_{45} \cdot d_{46} \cdot d_{47} \cdot d_{48} \cdot d_{49} \cdot d_{50} \cdot d_{51} \cdot d_{52} \cdot d_{53} \cdot d_{54} \cdot d_{55} \cdot d_{56} \cdot d_{57} \cdot d_{58} \cdot d_{59} \cdot d_{60} \cdot d_{61} \cdot d_{62} \cdot d_{63} \cdot d_{64} \cdot d_{65} \cdot d_{66} \cdot d_{67} \cdot d_{68} \cdot d_{69} \cdot d_{70} \cdot d_{71} \cdot d_{72} \cdot d_{73} \cdot d_{74} \cdot d_{75} \cdot d_{76} \cdot d_{77} \cdot d_{78} \cdot d_{79} \cdot d_{80} \cdot d_{81} \cdot d_{82} \cdot d_{83} \cdot d_{84} \cdot d_{85} \cdot d_{86} \cdot d_{87} \cdot d_{88} \cdot d_{89} \cdot d_{90} \cdot d_{91} \cdot d_{92} \cdot d_{93} \cdot d_{94} \cdot d_{95} \cdot d_{96} \cdot d_{97} \cdot d_{98} \cdot d_{99} \cdot d_{100} \cdot d_{101} \cdot d_{102} \cdot d_{103} \cdot d_{104} \cdot d_{105} \cdot d_{106} \cdot d_{107} \cdot d_{108} \cdot d_{109} \cdot d_{110} \cdot d_{111} \cdot d_{112} \cdot d_{113} \cdot d_{114} \cdot d_{115} \cdot d_{116} \cdot d_{117} \cdot d_{118} \cdot d_{119} \cdot d_{120} \cdot d_{121} \cdot d_{122} \cdot d_{123} \cdot d_{124} \cdot d_{125} \cdot d_{126} \cdot d_{127} \cdot d_{128} \cdot d_{129} \cdot d_{130} \cdot d_{131} \cdot d_{132} \cdot d_{133} \cdot d_{134} \cdot d_{135} \cdot d_{136} \cdot d_{137} \cdot d_{138} \cdot d_{139} \cdot d_{140} \cdot d_{141} \cdot d_{142} \cdot d_{143} \cdot d_{144} \cdot d_{145} \cdot d_{146} \cdot d_{147} \cdot d_{148} \cdot d_{149} \cdot d_{150} \cdot d_{151} \cdot d_{152} \cdot d_{153} \cdot d_{154} \cdot d_{155} \cdot d_{156} \cdot d_{157} \cdot d_{158} \cdot d_{159} \cdot d_{160} \cdot d_{161} \cdot d_{162} \cdot d_{163} \cdot d_{164} \cdot d_{165} \cdot d_{166} \cdot d_{167} \cdot d_{168} \cdot d_{169} \cdot d_{170} \cdot d_{171} \cdot d_{172} \cdot d_{173} \cdot d_{174} \cdot d_{175} \cdot d_{176} \cdot d_{177} \cdot d_{178} \cdot d_{179} \cdot d_{180} \cdot d_{181} \cdot d_{182} \cdot d_{183} \cdot d_{184} \cdot d_{185} \cdot d_{186} \cdot d_{187} \cdot d_{188} \cdot d_{189} \cdot d_{190} \cdot d_{191} \cdot d_{192} \cdot d_{193} \cdot d_{194} \cdot d_{195} \cdot d_{196} \cdot d_{197} \cdot d_{198} \cdot d_{199} \cdot d_{200} \cdot d_{201} \cdot d_{202} \cdot d_{203} \cdot d_{204} \cdot d_{205} \cdot d_{206} \cdot d_{207} \cdot d_{208} \cdot d_{209} \cdot d_{210} \cdot d_{211} \cdot d_{212} \cdot d_{213} \cdot d_{214} \cdot d_{215} \cdot d_{216} \cdot d_{217} \cdot d_{218} \cdot d_{219} \cdot d_{220} \cdot d_{221} \cdot d_{222} \cdot d_{223} \cdot d_{224} \cdot d_{225} \cdot d_{226} \cdot d_{227} \cdot d_{228} \cdot d_{229} \cdot d_{230} \cdot d_{231} \cdot d_{232} \cdot d_{233} \cdot d_{234} \cdot d_{235} \cdot d_{236} \cdot d_{237} \cdot d_{238} \cdot d_{239} \cdot d_{240} \cdot d_{241} \cdot d_{242} \cdot d_{243} \cdot d_{244} \cdot d_{245} \cdot d_{246} \cdot d_{247} \cdot d_{248} \cdot d_{249} \cdot d_{250} \cdot d_{251} \cdot d_{252} \cdot d_{253} \cdot d_{254} \cdot d_{255} \cdot d_{256} \cdot d_{257} \cdot d_{258} \cdot d_{259} \cdot d_{260} \cdot d_{261} \cdot d_{262} \cdot d_{263} \cdot d_{264} \cdot d_{265} \cdot d_{266} \cdot d_{267} \cdot d_{268} \cdot d_{269} \cdot d_{270} \cdot d_{271} \cdot d_{272} \cdot d_{273} \cdot d_{274} \cdot d_{275} \cdot d_{276} \cdot d_{277} \cdot d_{278} \cdot d_{279} \cdot d_{280} \cdot d_{281} \cdot d_{282} \cdot d_{283} \cdot d_{284} \cdot d_{285} \cdot d_{286} \cdot d_{287} \cdot d_{288} \cdot d_{289} \cdot d_{290} \cdot d_{291} \cdot d_{292} \cdot d_{293} \cdot d_{294} \cdot d_{295} \cdot d_{296} \cdot d_{297} \cdot d_{298} \cdot d_{299} \cdot d_{300} \cdot d_{301} \cdot d_{302} \cdot d_{303} \cdot d_{304} \cdot d_{305} \cdot d_{306} \cdot d_{307} \cdot d_{308} \cdot d_{309} \cdot d_{310} \cdot d_{311} \cdot d_{312} \cdot d_{313} \cdot d_{314} \cdot d_{315} \cdot d_{316} \cdot d_{317} \cdot d_{318} \cdot d_{319} \cdot d_{320} \cdot d_{321} \cdot d_{322} \cdot d_{323} \cdot d_{324} \cdot d_{325} \cdot d_{326} \cdot d_{327} \cdot d_{328} \cdot d_{329} \cdot d_{330} \cdot d_{331} \cdot d_{332} \cdot d_{333} \cdot d_{334} \cdot d_{335} \cdot d_{336} \cdot d_{337} \cdot d_{338} \cdot d_{339} \cdot d_{340} \cdot d_{341} \cdot d_{342} \cdot d_{343} \cdot d_{344} \cdot d_{345} \cdot d_{346} \cdot d_{347} \cdot d_{348} \cdot d_{349} \cdot d_{350} \cdot d_{351} \cdot d_{352} \cdot d_{353} \cdot d_{354} \cdot d_{355} \cdot d_{356} \cdot d_{357} \cdot d_{358} \cdot d_{359} \cdot d_{360} \cdot d_{361} \cdot d_{362} \cdot d_{363} \cdot d_{364} \cdot d_{365} \cdot d_{366} \cdot d_{367} \cdot d_{368} \cdot d_{369} \cdot d_{370} \cdot d_{371} \cdot d_{372} \cdot d_{373} \cdot d_{374} \cdot d_{375} \cdot d_{376} \cdot d_{377} \cdot d_{378} \cdot d_{379} \cdot d_{380} \cdot d_{381} \cdot d_{382} \cdot d_{383} \cdot d_{384} \cdot d_{385} \cdot d_{386} \cdot d_{387} \cdot d_{388} \cdot d_{389} \cdot d_{390} \cdot d_{391} \cdot d_{392} \cdot d_{393} \cdot d_{394} \cdot d_{395} \cdot d_{396} \cdot d_{397} \cdot d_{398} \cdot d_{399} \cdot d_{400} \cdot d_{401} \cdot d_{402} \cdot d_{403} \cdot d_{404} \cdot d_{405} \cdot d_{406} \cdot d_{407} \cdot d_{408} \cdot d_{409} \cdot d_{410} \cdot d_{411} \cdot$$

Transformation between the *Judd* purity and chromaticity
 Data see K. Richter, PhD thesis, University of Basel (Switzerland), 1969, page 83.
 For the antagonistic spectral elementary colours
 $\lambda_B = 475$ nm, $\lambda_C = 502$ nm, $\lambda_Y = 574$ nm, $\lambda_R = 494$ nm
 modern coordinates $\bar{x}$, $\bar{y}$, $\bar{z}$ are used instead of $\bar{x}_1$ (1 to 3).
 Relation between opponent and spectral colour values:

$$\begin{pmatrix} \bar{x}(L) \\ \bar{y}(L) \\ \bar{z}(L) \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} i(L) \\ q(L) \\ d(L) \end{pmatrix} = \begin{pmatrix} 0.9093 & 0.3338 & -0.0133 \\ 1.0000 & 0.0000 & 0.0000 \\ 0.4924 & -0.0574 & -0.4136 \end{pmatrix} \begin{pmatrix} i(L) \\ q(L) \\ d(L) \end{pmatrix} \quad (2)$$

 The tristimulus values L , A , B and X , Y , Z are the same transformations.
 The normalized purity data a and b , are defined in LabMUN 1969 as follows:
 $a_x = n_x/n_0 = X-X_0/Y-Y_0$; $n_x = 2.8$ (3) $b_x = b_1 - b_2 = Z-Y/Z-Y_0$ (4) $x = 1 - x - y$ (5)
 Use of equations (2) to (5) leads to:

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} & b_{11} & b_{12} & b_{13} \\ a_{21} & a_{22} & a_{23} & b_{21} & b_{22} & b_{23} \\ a_{31} & a_{32} & a_{33} & b_{31} & b_{32} & b_{33} \end{pmatrix} \begin{pmatrix} L \\ A \\ B \end{pmatrix} \quad (6)$$

 with $x = X/(X+Y+Z)$ and $y = Y/(X+Y+Z)$ (7)

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} & b_{11} & b_{12} & b_{13} \\ a_{21} & a_{22} & a_{23} & b_{21} & b_{22} & b_{23} \\ a_{31} & a_{32} & a_{33} & b_{31} & b_{32} & b_{33} \end{pmatrix} \begin{pmatrix} L \\ A \\ B \end{pmatrix} \quad (8)$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} & b_{11} & b_{12} & b_{13} \\ a_{21} & a_{22} & a_{23} & b_{21} & b_{22} & b_{23} \\ a_{31} & a_{32} & a_{33} & b_{31} & b_{32} & b_{33} \end{pmatrix} \begin{pmatrix} L \\ A \\ B \end{pmatrix} \quad (9)$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} & b_{11} & b_{12} & b_{13} \\ a_{21} & a_{22} & a_{23} & b_{21} & b_{22} & b_{23} \\ a_{31} & a_{32} & a_{33} & b_{31} & b_{32} & b_{33} \end{pmatrix} \begin{pmatrix} L \\ A \\ B \end{pmatrix} \quad (10)$$

 (1,000+0,000+0,000) (2,3567+0,0987A-0,4269B) (1)

Judd spectral tristimulus and chromatic values $\bar{l}(\lambda)$, $\bar{a}(\lambda)$, $\bar{b}(\lambda)$
 Data see R. Kicher, PhD thesis, University of Basel (Switzerland), 1969, page 2

The equations between spectral onponent and tristimulus colour values a and b are:

$$\begin{pmatrix} \bar{l}(\lambda) \\ \bar{a}(\lambda) \\ \bar{b}(\lambda) \end{pmatrix} \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix} \begin{pmatrix} \bar{x}(\lambda) \\ \bar{y}(\lambda) \\ \bar{z}(\lambda) \end{pmatrix} = \begin{pmatrix} 0.0000 & 1.0000 & 0.0000 \\ 2.9797 & -2.6662 & -0.0960 \\ -0.4139 & 1.4571 & -2.4046 \end{pmatrix} \begin{pmatrix} \bar{x}(\lambda) \\ \bar{y}(\lambda) \\ \bar{z}(\lambda) \end{pmatrix}$$

The weighting constants in the RG (a) and YB (b) direction used are $n_1=1$, $n_2=1$.

503nm $\bar{a}(\lambda)=1$
 574nm $\bar{b}(\lambda)=-1$

$\bar{l}(\lambda)$
 $\bar{a}(\lambda)$ ($n_1=1$)
 $\bar{b}(\lambda)$ ($n_2=1$)

400 500 600 700 λ_c/nm

495 530 567 λ_c/nm

TUB-test chart DE16; *JUDD* observer, colorimetric calculations of chromatic values
Data see *K. Richter*, PhD thesis, University of Basel (Switzerland), 1969

TUB material: code=rha4ta