Transformation between the *Judd* purity and chromaticity

Data see *K. Richter*, PhD thesis, University of Basel (Switzerland), 1969, page 83.

For the antagonistic spectral elementary colours
λ_B= 475 nm, λ_G= 502 nm, λ_Y= 574 nm, λ_R= 494c nm
modern coordinates $\bar{l}$, $\bar{a}$, b are used instead of $\bar{x}_i$ (i=1 to 3).

Relation between opponent and spectral colour values:

$$\begin{pmatrix}\bar{x}(\lambda)\\ \bar{y}(\lambda)\\ \bar{z}(\lambda)\end{pmatrix}=\begin{pmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{pmatrix}\cdot\begin{pmatrix}\bar{l}(\lambda)\\ \bar{a}(\lambda)\\ b(\lambda)\end{pmatrix}=\begin{pmatrix}0{,}9093 & 0{,}3338 & -0{,}0133\\ 1{,}0000 & 0{,}0000 & 0{,}0000\\ 0{,}4494 & -0{,}0574 & -0{,}4136\end{pmatrix}\cdot\begin{pmatrix}\bar{l}(\lambda)\\ \bar{a}(\lambda)\\ b(\lambda)\end{pmatrix} \quad (2)$$

The tristimulus values *L, A, B* and *X, Y, Z* need the same transformations.
The normalized purity data a_n and b_n are defined in LabMUN 1969 as follows:
$a_n=n_a a_u=n_a X/Y=n_a x/y$; n_a=2,8 (3) $b_n=b_u=-Z/Y=z/y$ (4) $z=1-x-y$ (5)
Use of equations (2) to (5) leads to:

$$\begin{pmatrix}X\\ Y\\ Z\end{pmatrix}=\begin{pmatrix}a_{11} & a_{12}\,a_n/n_a & a_{13}\,b_n\\ a_{21} & a_{22}\,a_n/n_a & a_{23}\,b_n\\ a_{31} & a_{32}\,a_n/n_a & a_{33}\,b_n\end{pmatrix}\cdot\begin{pmatrix}L\\ L\\ L\end{pmatrix} \quad (6) \quad \text{with } x=X/(X+Y+Z) \text{ and } y=Y/(X+Y+Z) \quad (7)$$

$$x=[a_{11}+a_{12}a_n/n_a+a_{13}b_n]/[a_{11}+a_{21}+a_{31}+(a_{12}+a_{22}+a_{32})/n_a+(a_{13}+a_{23}+a_{33})b_n]$$
$$y=[a_{21}+a_{22}a_n/n_a+a_{23}b_n]/[a_{11}+a_{21}+a_{31}(a_{12}+a_{22}+a_{32})/n_a+(a_{13}+a_{23}+a_{33})b_n] \quad (8)$$

$$x=(\alpha_{11}+\alpha_{12}a_n+\alpha_{13}b_n)/(\alpha_{31}+\alpha_{32}a_n+\alpha_{33}b_n)$$
$$=(0{,}9093+0{,}1192a_n-0{,}0133b_n)/(2{,}3587+0{,}0987a_n-0{,}4269b_n) \quad (9)$$

$$y=(\alpha_{21}+\alpha_{22}a_n+\alpha_{23}b_n)/(\alpha_{31}+\alpha_{32}a_n+\alpha_{33}b_n)$$
$$=(1{,}000+0{,}000a_n+0{,}000b_n)/(2{,}3587+0{,}0987a_n-0{,}4269b_n) \quad (10)$$