Transformation between the *Judd* tristimulus and opponent values

Data see *K. Richter*, PhD thesis, University of Basel (Switzerland), 1969, page 81.

For the antagonistic spectral elementary colours
λ_B= 475 nm, $\lambda_{G\subset}$= 502 nm, λ_Y= 574 nm, λ_R= 494c nm
the coordinates $\bar{x}_i$ (i=1 to 3) are used instead of modern coordinates $\bar{l}$, $\bar{a}$, $\bar{b}$.

Linear model equations between spectral colour values in both directions:

$$\begin{pmatrix}\bar{x}_l(\lambda)\\ \bar{x}_a(\lambda)\\ \bar{x}_b(\lambda)\end{pmatrix}=\begin{pmatrix}b_{11} & b_{12} & b_{13}\\ b_{21} & b_{22} & b_{23}\\ b_{31} & b_{32} & b_{33}\end{pmatrix}\cdot\begin{pmatrix}\bar{x}(\lambda)\\ \bar{y}(\lambda)\\ \bar{z}(\lambda)\end{pmatrix}=\begin{pmatrix}0{,}0000 & 1{,}0000 & 0{,}0000\\ 2{,}9797 & -2{,}6662 & -0{,}0960\\ -0{,}4139 & 1{,}4571 & -2{,}4046\end{pmatrix}\cdot\begin{pmatrix}\bar{x}(\lambda)\\ \bar{y}(\lambda)\\ \bar{z}(\lambda)\end{pmatrix} \quad (1)$$

$$\begin{pmatrix}\bar{x}(\lambda)\\ \bar{y}(\lambda)\\ \bar{z}(\lambda)\end{pmatrix}=\begin{pmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{pmatrix}\cdot\begin{pmatrix}\bar{x}_l(\lambda)\\ \bar{x}_a(\lambda)\\ \bar{x}_b(\lambda)\end{pmatrix}=\begin{pmatrix}0{,}9093 & 0{,}3338 & -0{,}0133\\ 1{,}0000 & 0{,}0000 & 0{,}0000\\ 0{,}4494 & -0{,}0574 & -0{,}4136\end{pmatrix}\cdot\begin{pmatrix}\bar{x}_l(\lambda)\\ \bar{x}_a(\lambda)\\ \bar{x}_b(\lambda)\end{pmatrix} \quad (2)$$

The tristimulus values X_b X_a X_l and X, Y, Z need the same transformations.

The unnormalized purity data $a_u$and $b_u$are defined in LabMUN 1969 as follows:
$a_u=X_a/X_l=x_a/x_l$ (3) $b_u=X_b/X_l=x_b/x_l$ (4) $x_{\subset}=1-x_{\subset}-x_l$ (5)

The unnormalized purity data $a_u$and $b_u$are defined in LabMUN 1969 as follows:
$a_u=[(b_{21}-b_{23})x+(b_{22}-b_{23})y+b_{23}]/y$
=(3,0757x−2,5702y−0,0960)/y (6)

$b_u=[(b_{31}-b_{33})x+(b_{32}-b_{33})y+b_{33}]/y$
=(1,9906x+3,8617y−2,4046)/y (7)

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=2,8(3,0757x−2,5702y−0,0960)/y
=(8,6119x−7,1965y−0,2688)/y (6)

$b_u=[(b_{31}-b_{33})x+(b_{32}-b_{33})y+b_{33}]/y$
=(1,9906x+3,8617y−2,4046)/y (7)

Transformation between the *Judd* purity and chromaticity

Data see *K. Richter*, PhD thesis, University of Basel (Switzerland), 1969, page 83.

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Relation between opponent and spectral colour values:

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$$x=[a_{11}+a_{12}a_u+a_{13}b_u]/[a_{11}+a_{21}+a_{31}+(a_{12}+a_{22}+a_{32})a_u+(a_{13}+a_{23}+a_{33})b_u] \quad (8)$$
$$y=[a_{21}+a_{22}a_u+a_{23}b_u]/[a_{11}+a_{21}+a_{31}+(a_{12}+a_{22}+a_{32})a_u+(a_{13}+a_{23}+a_{33})b_u]$$

$$x=(a_{11}+a_{12}a_u+a_{13}b_u)/(a_{31}+a_{32}a_u+a_{33}b_u)$$
$$=(0{,}9093+0{,}3338a_u-0{,}0133b_u)/(2{,}3587+0{,}2764a_u-0{,}4269b_u) \quad (9)$$
$$y=(a_{21}+a_{22}a_u+a_{23}b_u)/(a_{31}+a_{32}a_u+a_{33}b_u)$$
$$=(1{,}000+0{,}000a_u+0{,}000b_u)/(2{,}3587+0{,}2764a_u-0{,}4269b_u) \quad (10)$$

Transformation between the *Judd* purity and chromaticity

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