Transformation between the *Judd* purity and chromaticity

Data see *K. Richter*, PhD thesis, University of Basel (Switzerland), 1969, page 83.

For the antagonistic spectral elementary colours
λ_B= 475 nm, λ_G= 502 nm, λ_Y= 574 nm, λ_R= 494c nm
the coordinates $\bar{x}_i$ (i=1 to 3) are used instead of modern coordinates $\bar{l}$, $\bar{a}$, $\bar{b}$.

Relation between opponent and spectral colour values:

$$\begin{pmatrix}\bar{x}(\lambda)\\ \bar{y}(\lambda)\\ \bar{z}(\lambda)\end{pmatrix}=\begin{pmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{pmatrix}\cdot\begin{pmatrix}\bar{x}_1(\lambda)\\ \bar{x}_2(\lambda)\\ \bar{x}_3(\lambda)\end{pmatrix}=\begin{pmatrix}0{,}9093 & 0{,}3338 & -0{,}0133\\ 1{,}0000 & 0{,}0000 & 0{,}0000\\ 0{,}4494 & -0{,}0574 & -0{,}4136\end{pmatrix}\cdot\begin{pmatrix}\bar{x}_1(\lambda)\\ \bar{x}_2(\lambda)\\ \bar{x}_3(\lambda)\end{pmatrix} \quad (2)$$

The tristimulus values X_1, X_2, X_3 and X, Y, Z need the same transformations.
The unnormalized purity data a_u and b_u are defined in LabMUN 1969 as follows:

$a_u=X_2/X_1=x_2/x_1$ (3) $\quad b_u=X_3/X_1=x_3/x_1$ (4) $\quad x_3=1-x_2-x_1$ (5)

Use of equations (2) to (5) leads to:

$$\begin{pmatrix}X\\ Y\\ Z\end{pmatrix}=\begin{pmatrix}a_{11} & a_{12}a_u & a_{13}b_u\\ a_{21} & a_{22}a_u & a_{23}b_u\\ a_{31} & a_{32}a_u & a_{33}b_u\end{pmatrix}\cdot\begin{pmatrix}X_1\\ X_1\\ X_1\end{pmatrix} \quad (6)$$ and with $x=X/(X+Y+Z)$ and $y=Y/(X+Y+Z)$ (7)

$$x=[a_{11}+a_{12}a_u+a_{13}b_u]/[a_{11}+a_{21}+a_{31}+(a_{12}+a_{22}+a_{32})a_u+(a_{13}+a_{23}+a_{33})b_u]$$
$$y=[a_{21}+a_{22}a_u+a_{23}b_u]/[a_{11}+a_{21}+a_{31}+(a_{12}+a_{22}+a_{32})a_u+(a_{13}+a_{23}+a_{33})b_u] \quad (8)$$

$$x=(\alpha_{11}+\alpha_{12}a_u+\alpha_{13}b_u)/(\alpha_{31}+\alpha_{32}a_u+\alpha_{33}b_u)$$
$$=(0{,}9093+0{,}3338a_u-0{,}0133b_u)/(2{,}3587+0{,}2764a_u-0{,}4269b_u) \quad (9)$$

$$y=(\alpha_{21}+\alpha_{22}a_u+\alpha_{23}b_u)/(\alpha_{31}+\alpha_{32}a_u+\alpha_{33}b_u)$$
$$=(1{,}000+0{,}000a_u+0{,}000b_u)/(2{,}3587+0{,}2764a_u-0{,}4269b_u) \quad (10)$$