

Weber-Fechner law in CIE 230:2019 for threshold colour differences of surface colours

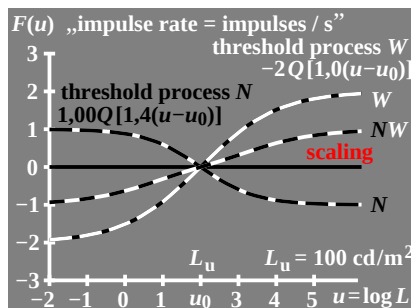
The Weber-Fechner law describes the lightness L^* as *logarithmic* function of L_u .
The Stevens law describes the lightness L^* as *potential* function of $L_u = 3/5$.
IEC 61966-2-1 uses a similar potential function $L^*_{IEC} = m L_u^{1/2.4}$.
The Weber-Fechner law is equivalent to the equation: $\Delta L_u = c L_u$ [1]
Integration leads to the logarithmic equation: $L^* = k \log(L_u)$ [2]
Derivation leads for $\Delta L_u = 1$ to the linear equation: $L^*_{\Delta L_u=1} = k_0$ ($k_0 = 46$, $k_1 = 63$) [3]
For colours in offices the standard contrast range is 25:1=90:3.6.

Table 1: CIE tristimulus value Y, luminance L_u , and lightness L^*

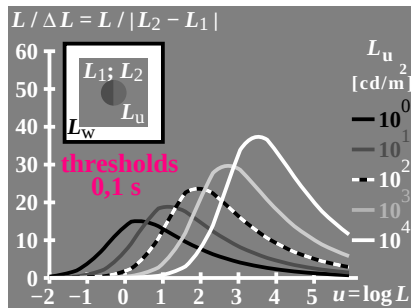
Colour (matte)	Tristimulus value Y	office luminance L_u [cd/m ²]	relative luminance L_u/L_u	CIE lightness L^*	relative lightness L^*/L^*
(contrast) (25:1=90:3.6)	Y	L [cd/m ²]	L_u/L_u	$L^*_{\Delta L_u=1}$	L^*/L^*
White W (paper)	90	142	=28.2*5	94	=50+44
Grey Z (paper)	18	28.2	1	50	=k log(5)
Black N (paper)	3.6	5.6	0.2	18	=k log(1)
Black N (paper)	=18/5	28.2/5		50-32	=k log(0.2)

For the lightness range between L^*_{-40} and 40 the constant is: $k=40/\log(5)=57$

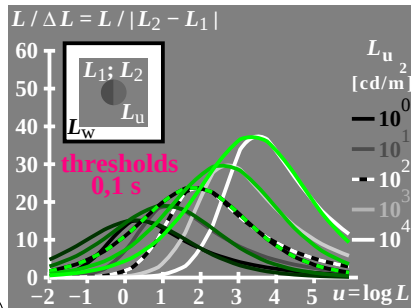
AET00-1N



AET00-3N



AET00-5N



AET00-7N

Weber-Fechner law in CIE 230:2019 for threshold colour differences of surface colours and two ranges $0.2 \leq L_u \leq 1$ and $1 < L_u < 5$

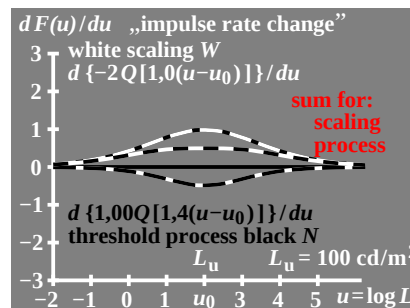
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IEC 61966-2-1 uses a similar potential function $L^*_{IEC} = m L_u^{1/2.4}$.
The Weber-Fechner law is equivalent to the linear equation: $\Delta L_u = c L_u$ ($c=0.1$) [1]
Integration leads to the logarithmic equation: $L^* = k \log(L_u)$ ($c=0.1$) [2]
Derivation leads for $\Delta L_u = 1$ to the linear equation: $L^*_{\Delta L_u=1} = k_0$ ($k_0 = 46$, $k_1 = 63$) [3]
For colours in offices the standard contrast range is 25:1=90:3.6.

Table 1: CIE tristimulus value Y, luminance L_u , and lightness L^*

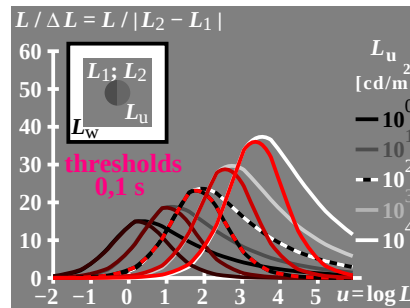
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For the two lightness ranges it is $k_0 = -32/\log(0.2) = 46$ and $k_1 = 44/\log(5) = 63$.

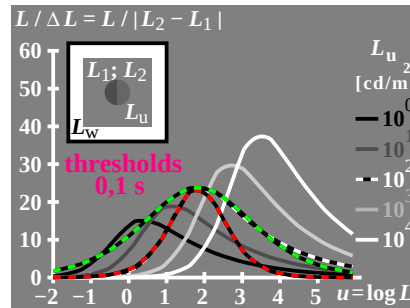
AET00-2N



AET00-4N



AET00-6N



AET00-8N

line element of Stiles (1946) with „color values” L_P, M_D, S_T

three separate color signal functions

$F(L_P) = i \ln(1 + 9 L_P)$
 $F(M_D) = j \ln(1 + 9 M_D)$
 $F(S_T) = k \ln(1 + 9 S_T)$

Taylor-derivations:

$\Delta F(L_P, M_D, S_T) = \frac{dF}{dL_P} \Delta L_P + \frac{dF}{dM_D} \Delta M_D + \frac{dF}{dS_T} \Delta S_T$

$= \frac{9i}{1+9L_P} \Delta L_P + \frac{9j}{1+9M_D} \Delta M_D + \frac{9k}{1+9S_T} \Delta S_T$

AET01-1N

functions $q[k(u-u_0)]$

„achromatic signal”-description

with $u = \log L$ (L = luminance)
 $u_0 = \log L_u$ (L_u = surround luminance)

$q[k(u-u_0)] = 1 + 1/[1 + \sqrt{2} e^{k(u-u_0)}]$

function values:

$q[k(u-u_0) \rightarrow +\infty] = 1$
 $q[k(u-u_0) = 0] = \sqrt{2}$
 $q[k(u-u_0) \rightarrow -\infty] = 2$

AET01-3N

„achromatic signal” discrimination as function of relative light density

$h = \ln H = k(u-u_0)$, $\ln$ = natural log.

$Q' = \frac{d}{dH} [\ln\{1 + 1/(1 + \sqrt{2} H)\}]/\ln \sqrt{2}$

$= -\sqrt{2}/[\ln \sqrt{2} (1 + \sqrt{2} H)(2 + \sqrt{2} H)]$

function values:

$Q'[k(u-u_0) \rightarrow +\infty] = 0$
 $Q'[k(u-u_0) = 0] = -0,5$
 $Q'[k(u-u_0) \rightarrow -\infty] = 0$

AET01-5N

double line element of Richter (1987) for the lighting technology with the luminance $L = f(L_P, M_D, S_T)$

$F(L) = \int_{-\infty}^L (L/\Delta L) dL$ (relative lightness?)
 $F(L) = iQ(H) = \begin{cases} iQ(H) & (u < u_0) \\ iQ(\bar{H}) & (u \geq u_0) \end{cases}$

with: $k=1,4$ $\bar{k}=1$ $i=1$ $\bar{i}=-2$
 $u = \log L$ $u_0 = \log L_u$
 $H = e^{k(u-u_0)}$ $\bar{H} = e^{\bar{k}(u-u_0)}$

AET01-7N

line element of Vos&Walraven (1972) with „color values” L_P, M_D, S_T

three separate color signal functions

$F(L_P) = -2i\sqrt{L_P}$
 $F(M_D) = -2j\sqrt{M_D}$
 $F(S_T) = -2k\sqrt{S_T}$

Taylor-derivations:

$\Delta F(L_P, M_D, S_T) = \frac{dF}{dL_P} \Delta L_P + \frac{dF}{dM_D} \Delta M_D + \frac{dF}{dS_T} \Delta S_T$

$\Delta F(L_P, M_D, S_T) = \frac{i}{\sqrt{L_P}} \Delta L_P + \frac{j}{\sqrt{M_D}} \Delta M_D + \frac{k}{\sqrt{S_T}} \Delta S_T$

AET01-2N

„achromatic signal”-description functions $Q_{1m}[k(u-u_0)]$

with $u = \log L$ (L = luminance)
 $u_0 = \log L_u$ (L_u = surround luminance)

$Q_{1m}[k(u-u_0)] = \frac{1}{\ln \sqrt{2}} \ln q[k(u-u_0)] - m$

function values with $l = m = 1$:

$Q[k(u-u_0) \rightarrow +\infty] = 1$
 $Q[k(u-u_0) = 0] = 0$
 $Q[k(u-u_0) \rightarrow -\infty] = -1$

AET01-4N

luminance discrimination possibility $L/\Delta L$ as function of H

with: $L = 10^u$ $H = e^h = 10^{\log e k(u-u_0)}$
 $dL/du = \ln 10 L$ $dH/du = k H$

it follows: $L/\Delta L = [kH/(dH \ln 10)]$

$\frac{L}{\Delta L} = \text{const } H/[(1 + \sqrt{2} H)(2 + \sqrt{2} H)]$

function values:

$Q'[k(u-u_0) \rightarrow +\infty] = 0$
 $Q'[k(u-u_0) = 0] = \text{maximum}$
 $Q'[k(u-u_0) \rightarrow -\infty] = 0$

AET01-6N

double line element of Richter (1987) for the lighting technology with the luminance $L = f(L_P, M_D, S_T)$

$F(L) = \int_{-\infty}^L (L/\Delta L) dL$ (relative lightness?)
 $F(L) = iQ(H)$ $H = e^{k(u-u_0)}$
 $Q(H) = [\ln\{1 + 1/(1 + \sqrt{2} H)\}]/\ln \sqrt{2} - 1$

Taylor-derivations:

$\Delta F(L) = \frac{dF}{dL} \Delta L = i \frac{dQ}{dH} \Delta H$

$= -i\sqrt{2} \Delta H/[\ln \sqrt{2} (1 + \sqrt{2} H)(2 + \sqrt{2} H)]$

AET01-8N