Basic data, methods and formula to bridge the gap for color differences

between large CIELAB color differences, e.g. DE*ab=20, and threshold differences for office conditions

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Abstract

Some basic results of a paper “Cube root color spaces and chromatic adaptation” published in 1980 in Color, Research and Application and new experimental data and models are useful to bridge the gap between threshold and large CIELAB colour differences.

It is the aim of the new model not to change the main properties of the CIELAB colour space which is recommended and used for large colour differences in the colour reproduction area nor the main properties of the CIE colour difference formulas which are recommended for up to 5 CIELAB units and based on super threshold (pass-fail) experiments. The improved model is based on experimental threshold results of e. g. MacAdam, Richter, and Holtsmark-Valberg and others.

The metric to describe colour threshold and colour scaling data is very different. The first is nearly a linear metric and the second at least in the blue yellow direction a very nonlinear metric. Main properties of the threshold metric are symmetric for complementary optimal colours which will lead to a special structure of the threshold colour difference formula.

1.0 Complementary optimal colours and Holtsmark-Valberg threshold results

In 1969 Holtsmark and Valberg studied the colour thresholds of complementary optimal colours. Such complementary optimal are known from the study of black and white borders studied by the German poet Goethe around 1800.

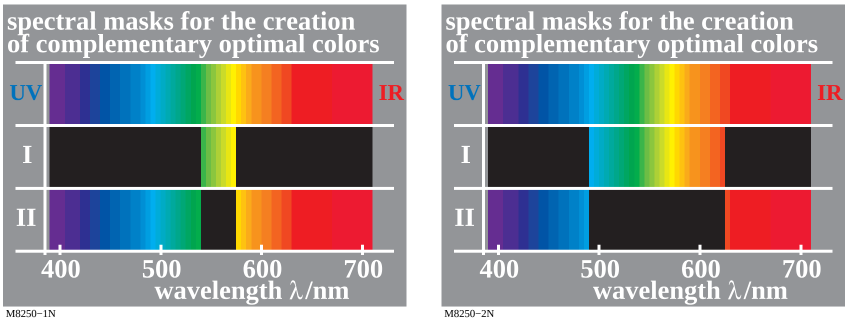

Figure 1: Spectrum, complementary optimal colours and the Valberg–Holtsmark thresholds

Fig. 1 shows small band (dark) and large band (light) optimal colours (part I) and large band (light) and small band (dark) complementary optimal colours (part II). The Holtsmark and Valberg results show the threshold discrimination for complementary optimal colours. The discrimination at threshold is the same. We have to explain this by colorimetric calculations and physiological models, for an example see (5.000 kByte)

http://www.ps.bam.de/CIE63/HV01.PDF

This results define a **symmetric structure for a colour threshold formula** which must calculate the same difference at least for the hue discrimination which is here the main change in the experiments

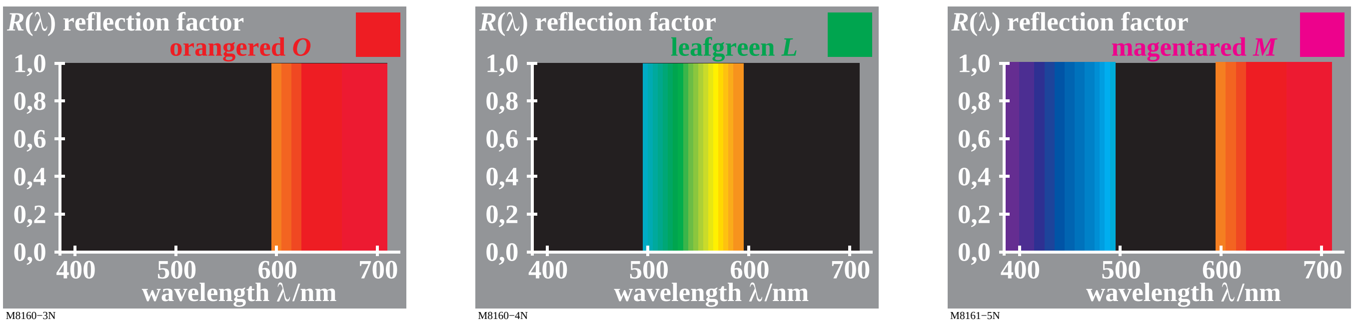

Figure 2: Optimal colour Orangered O, Leafgreen L and Magentared M

Fig. 3 shows three optimal colours. Leafgreen and Magentared are complementary. The names are defined in ISO/IEC 15775. The abbreviations RGB for the primaries are avoided and reserved for the unique hues RGBJ. The optimal colour Y is slightly greenish compared to the unique hue Yellow J (J=french jeaunne = yellow). The two complementary stimuli Leafgreen and Magentared mix to the stimuli of the whole spectrum which is white W.

> Remark 1: The change of the reflection for a good reproduction system is near 490 nm and 590 nm. The following discussions and results are not dependent on the exact choice of these wavelength, e. g. 500 nm and 600 nm is also possible. But the data in Fig. 3 will slightly change.
>
> Remark 2: The basic colours CMYOLV of a real reproduction system will differ from this ideal case

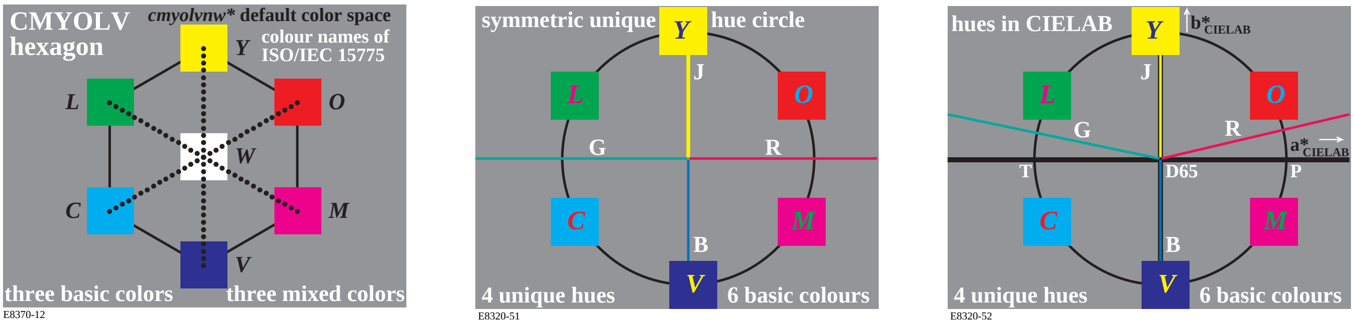

Figure 3: Six basic colours and four unique hues and order in CIELAB

Fig. 3 shows on the left side the hexagon of six chromatic colours used in image technology. e. g. ISO/IEC 15775. There is the user requirement that the output device must produces 16 equally spaced steps between white and CMYOLV in CIELAB for digital colour device data in the file which are linearly spaced between 0 and 1. We will later normalize the CIE data between 0 and 1 which will produce data between 1 and 0 for the complementary colours.

In the middle of Fig. 3 the symmetric unique hue circle is shown. On the right side the order of the 6 basic colours and of the unique hues is shown in CIELAB. One must realize that both the unique hue Red R and Green G are shifted by about 15 degrees to the yellow. A stimuli mixture of a unique red and green will therefore produce a yellowish colour. A stimuli mixture of a bluish red (Purple) and a bluish green (Turquoise) will produce an achromatic colour. We will study later the thresholds along the colour series produced by a stimuli mixture of P and T.

We study in the following the case that the six colorants are optimal colours with reflection factors of either 1 or 0 (see Fig. 2). The CIE data are shown in Fig. 4. There is a special need here to choose the optimal colours. Holtsmark and Valberg have studied colour thresholds for optimal colours.

basic and mixed additive optimal colors for illuminant D65					
basic color or mixed color and name	CIE standard chromaticity x	y	CIE standard tristimulus value X	Y	Z
three additive basic optimal colors:					
O orangered	0,6695	0,3302	42,65	21,04	0,02
L leafgreen	0,2991	0,6351	34,87	74,04	7,67
V violetblue	0,1445	0,0393	18,06	4,90	102,02
three additive mixed optimal colors:					
C cyanblue	0,2191	0,3268	52,94	78,96	109,70
M magentared	0,3218	0,1375	60,73	25,95	102,04
Y yellow	0,4300	0,5274	77,53	95,09	7,69
D65 (white)	0,3131	0,3275	95,60	100,00	109,71

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basic and mixed additive optimal colors 01 normalized for illuminant D65					
basic color or mixed color and name	Range 01 normalized chromaticity x_{01}	y_{01}	Range 01 normalized tristimulus value $X_{01}=X/X_n$	$Y_{01}=Y/Y_n$	$Z_{01}=Z/Z_n$
three additive basic optimal colors:					
O orangered	0,6792	0,3304	0,4461	0,2105	0,0002
L leafgreen	0,3102	0,6295	0,3649	0,7405	0,0709
V violetblue	0,1620	0,0420	0,1890	0,0490	0,9289
three additive mixed optimal colors:					
C cyanblue	0,2364	0,3369	0,5539	0,7895	0,9998
M magentared	0,3479	0,1423	0,6351	0,2595	0,9291
Y yellow	0,4424	0,5188	0,8110	0,9510	0,0711
D65 (white)	0,3333	0,3333	1,0000	1,0000	1,0000

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Figure 4: Complementary optimal colour data and CIE (*x,y*) chromaticity diagram

Fig. 4 shows the CIE data and the normalisation for X, Y, and Z between 0 and 1 which is used in image technology

> Remark: For the calculation of $L^*a^*b^*$ of CIELAB the same normalisation is used by dividing X, Y, and Z by X_n, Y_n, and Z_n (see Fig. 5).

For any colour model one must have in mind the three levels of colour vision, the physical, the physiological and the psychological level. In Fig. 1 and 3 we see physical stimuli which will produce physiological effects which we will be study later. Fig. 3 shows some appearance attributes from the psychological level of vision, e. g. the symmetric unique hue circle and the order of the six basic colours within this hue circle. In the right part of Fig. 3 complementary stimuli data on two axis are combined with the order of the colours in a hue circle.

2.0 Holtsmark-Valberg threshold results and threshold formula for complementary optimal colours

The color metrics uses linear coordinate systems to describe the mixture of colours and nonlinear coordinate systems to describe the appearance of colours.

color valence metrics terms (color values: linear coordinates)		
color valence metric terms	**name and relationship with standard chromaticity values**	**notes:**
luminance factor	$Y = y\,(X + Y + Z)$	*definition in: CIEXYZ 1931*
chromatic value	*for linear chromatic value diagram (A, B)*	
red−green	$A = [X/Y - X_n/Y_n]\,Y = [a - a_n]\,Y$	*definition in:*
	$= [x/y - x_n/y_n]\,Y$	*opponent*
yellow−blue	$B = -0{,}4[Z/Y - Z_n/Y_n]\,Y = [b - b_n]\,Y$	*color system*
	$= -0{,}4[z/y - z_n/y_n]\,Y$	*n=D65 (surround)*
radial	$C = [A^2 + B^2]^{1/2}$	
saturation value	**= chromatic value / luminous value**	*definition in:*
red−green	$S_a = A/Y = X/Y - X_n/Y_n$	*opponent*
	$= x/y - x_n/y_n = a - a_n$	*color system*
yellow−blue	$S_b = B/Y = -0{,}4\,[Z/Y - Z_n/Y_n]$	
	$= -0{,}4[z/y - z_n/y_n] = b - b_n$	
radial	$S_c = C/Y$	
	$= [(a - a_n)^2 + (b - b_n)^2]^{1/2}$	
chromaticity value	*for linear chromaticity diagram (a, b)*	*definition in:*
red−green	$a = X/Y = x/y$	*opponent*
yellow−blue	$b = -0{,}4\,[Z/Y] = -0{,}4\,[z/y]$	*color system*
radial	$c = [(a - a_n)^2 + (b - b_n)^2]^{1/2}$	

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percieved color terms (colorness: cube root coordinates)		
percieved color terms	**name and relationship with standard chromaticity values**	**notes:**
lightness	$L^* = 116\,(Y/100)^{1/3} - 16$	*definition 1976 in:*
	Näherung: $L^* = 100\,(Y/100)^{1/3}$	*CIELAB for L*>8*
chromaticness	*for nonlinear chroma diagram (a*, b*)*	
red−green	$a^* = 500\,[(X/X_n)^{1/3} - (Y/Y_n)^{1/3}]$	*definition 1976 in:*
	$= 500\,(a' - a'_n)\,Y^{1/3}$	*CIELAB for L*>8*
yellow−blue	$b^* = 200\,[(Y/Y_n)^{1/3} - (Z/Z_n)^{1/3}]$	
	$= 500\,(b' - b'_n)\,Y^{1/3}$	*n=D65 (surround)*
radial	$C^* = [a^{*2} + b^{*2}]^{1/2}$	
saturation	**= chromaticness / lightness**	*definition*
red−green	$S^*_a = a^* / [100\,(Y/100)^{1/3}]$	*for:*
	$= 21{,}6\,(a' - a'_n)$	*CIELAB 1976*
yellow−blue	$S^*_b = b^* / [100\,(Y/100)^{1/3}]$	
	$= 21{,}6\,(b' - b'_n)$	
radial	$S^*_c = C^* / [100\,(Y/100)^{1/3}]$	
	$= 21{,}6\,[(a' - a'_n)^2 + (b' - b'_n)^2]^{1/2}$	
coordinates	*for nonlinear diagram (a', b')*	*definition in:*
red−green	$a' = (1/X_n)^{1/3}\,(x/y)^{1/3}$	*opponent*
	$= 0{,}2191\,(x/y)^{1/3}$ for *D65*	*color system*
yellow−blue	$b' = -0{,}4\,(1/Z_n)^{1/3}\,(z/y)^{1/3}$	
	$= -0{,}08376\,(z/y)^{1/3}$ for *D65*	
radial	$c' = [(a' - a'_n)^2 + (b' - b'_n)^2]^{1/2}$	

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Figure 5: Equations for the transfer of CIEXYZ data to other data in linear and nonlinear colour metrics

Fig. 5 shows main equation used in colour metrics. We will use the left part for the description of the Holtsmark-Valberg experiments by the **linear** equations. The red-green chromatic value for the basic colour is

$$A = (a - a_n)\,Y = (x/y - x_n/y_n)\,Y \qquad (2;1)$$

Then it is valid with the normalisation of image technology for the range 0 to 1 (Index 01)

$$A_{01} = (a_{01} - a_{01n})\,Y_{01} = (x_{01}/y_{01} - 1)\,Y_{01} = (X_{01}/Y_{01} - 1)\,Y_{01} = X_{01} - Y_{01}$$

For the complementary colours it is always valid $X_{01c} = 1 - X_{01}$, $Y_{01c} = 1 - Y_{01}$, $Z_{01c} = 1 - Z_{01}$. Therefore

$$A_{01c} = X_{01c} - Y_{01c} = 1 - X_{01} - (1 - Y_{01}) = Y_{01} - X_{01} = -A_1 \qquad (2;2)$$

If we use the three-dimensional difference in the linear space, then we have for the basic colours at threshold (th)

$$\text{delta } E^*_{ABY,th} = \{[\text{delta } A_{01}]^2 + [\text{delta } B_{01}]^2 + [\text{delta } Y_{01}]^2\}^{1/2} \qquad (2;3)$$

and for the complementary colours at threshold

$$\text{delta } E^*_{ABY,th,c} = \{[\text{delta } A_{01c}]^2 + [\text{delta } B_{01c}]^2 + [\text{delta } Y_{01c}]^2\}^{1/2} \qquad (2;4)$$

The absolute hue discrimination is for the complementary optimal colours the same because of equation (2;2)

$$A_{01c} = A_{01} \quad \text{and} \quad B_{01c} = B_{01} \qquad (2;5)$$

The last term delta Y_{01} is for the complementary colours different. If one colour is dark then the complementary is light. By the Weber-Fechner law it is valid for the achromatic discrimination along the luminance axis

$$\text{delta } Y_{01} = c_Y\,Y_{01} \qquad (2;6)$$

Therefore the above equations are only a solution for the special case that the luminance threshold is below the hue threshold. This is not always true in the Holtsmark-Valberg experiments because they report to see in some regions only a lightness difference. In this case we must look for a possibility to modify the threshold model.An possibility is to look at the **contrast sensivity**

$$Y_{01C} / (\text{delta } Y_{01C}) = Y_{01} / (\text{delta } Y_{01}) \qquad (2;7)$$

which is according to the Weber-Fechner law the **same for complementary colours**.

So instead of the equation (2;3) the following metric is in **complete agreement with the Holtsmark–Valberg threshold results for complementary optimal colours**

$$\text{delta } E^*_{ABY,th} = \{[\text{delta } A_{01}]^2 + [\text{delta } B_{01}]^2 + [(\text{delta } Y_{01})/Y_{01}]^2\}^{1/2} \qquad (2;8)$$

Basic data, methods and formula to bridge the gap for color differences

This formula will calculate in the colour space ABY at threshold the same value for complementary optimal colours

$$\text{delta } E^*_{ABY,th} = \text{delta } E^*_{ABY,th,c} \qquad (2;9)$$

Equation (2;8) may be the first equation which describes the surprising results of Holtsmark-Valberg for thresholds.

Remark 1: During the AIC-symposium in Soesterberg in 1971 there have been very controversial discussions about this results.

We must be careful about the interpretation of equation (2;8). This equation does not tell us at the moment how to scale A_{01}. In other words if

delta A_{01} = delta A_{01c} then it is also

(delta A_{01}) / A_{01} = (delta A_{01c}) / A_{01c}

The following speculative **equation for complementary optimal colours**

$$\text{delta } E^*_{ABY,th} = \{ [(\text{delta } A_{01}) / A_{01}]^2 + [(\text{delta } B_{01}) / B_{01}]^2 + [(\text{delta } Y_{01}) / Y_{01}]^2\}^{1/2} \qquad (2;10)$$

is also in **full agreement with the Holtsmark-Valberg results**.

Equations (2;8) and (2;10) are basic steps for the understanding. Many other experimental results on threshold will help us to decide on the best solution to describe threshold and probably scaling data.

3.0 Relationships between scaling and threshold

We have done scaling and threshold experiments with compensatory colours Violettblue – Yellow (V–Y) and Turquoise–Purple (T–P) approximately along the a* and b* axis of the CIELAB system

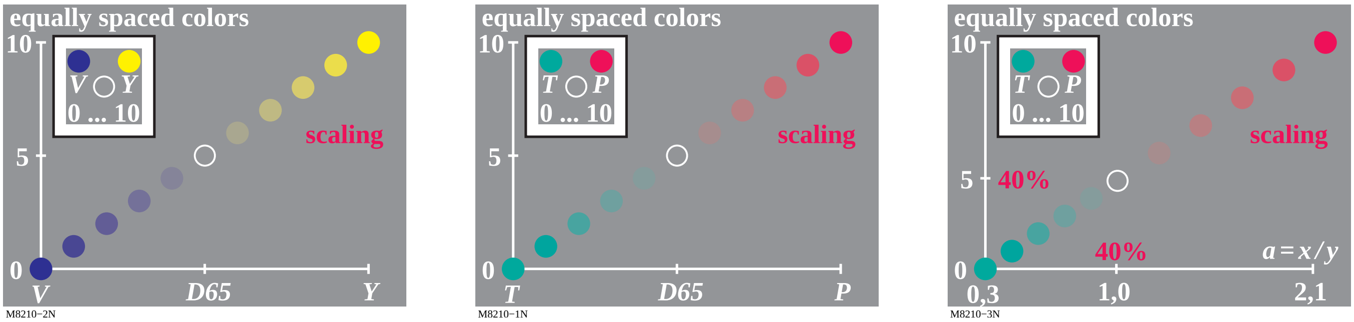

Figure 6: Experimental setup and results of scaling color series by interval scaling

The experimental setup is shown in the left top part of each figure. The luminance of the grey surround was 200 cd/m^2 and the white border about 1000 cd/m^2. The experimental setup was similar compared to 2 degree samples on paper and monitors in the offices. In the scaling experiments a interval scaling between the numbers 0 and 10 for the end colours and 5 for the mean grey was used. The Purple P was about 1.5 times more chromatic compared to the Turquoise T.

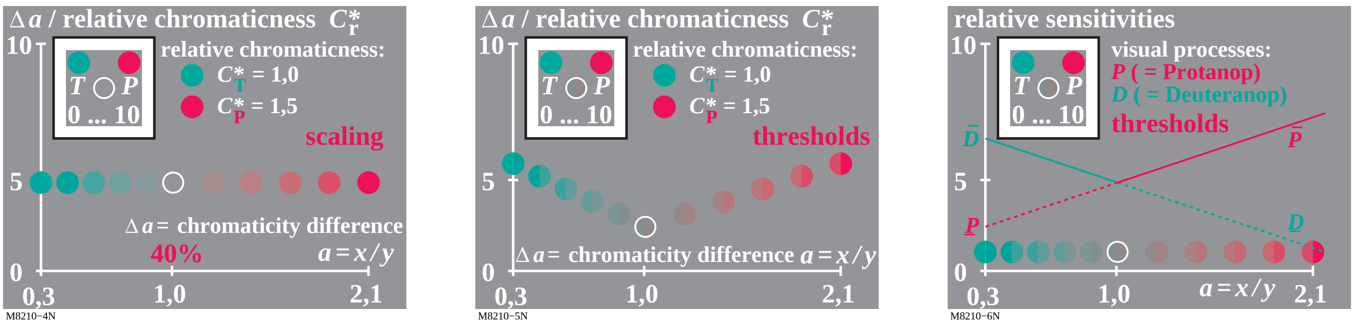

Figure 7: Experimental scaling results of a color series T – P and thresholds along this series

We calculate the differences in chromaticity (delta a) between equidistant steps and this chromaticity difference was constant. Therefore the chroma a* of the CIELAB system is proportional to the chromaticity a_{01}

$$a^*_{sc} \text{ proportional to } (a_{01} - a_{01n}) = (a_{01} - 1) \qquad (3;1)$$

In fact one can use the linear chromaticity *a* instead of the cube root of *a* (called *a'* in Fig. 5) multiplied with $Y_{01}^{1/3}$ to calculate *a**. The reason is the limited range of the chromaticity *a*. In this special limited range the linear and cube root equation are not too much different.

In the middle and right part threshold experiments have been done along the same colour series. The results are completely different, e. g. a solution to describe the two branches is (compare later, Fig. XX)

a^*_{th} proportional to $(a_{01} - 1)\ /\ (1 + 0.5\ |a_{01} - 1|\ )$ (3;2)

This solution is very similar to the change of chroma by the CMC colour difference formula and the CIE 2000 colour difference formula. Along the a^*_{CIELAB} axis it is defined

$a^*_{modified}$ proportional to $a^*_{CIELAB}\ /\ (1 + \text{const}\ a^*_{CIELAB})$ (3;3)

The above experiments have been done for an equal luminance level and further data with a change of the sample luminance must be included to get a more general model.

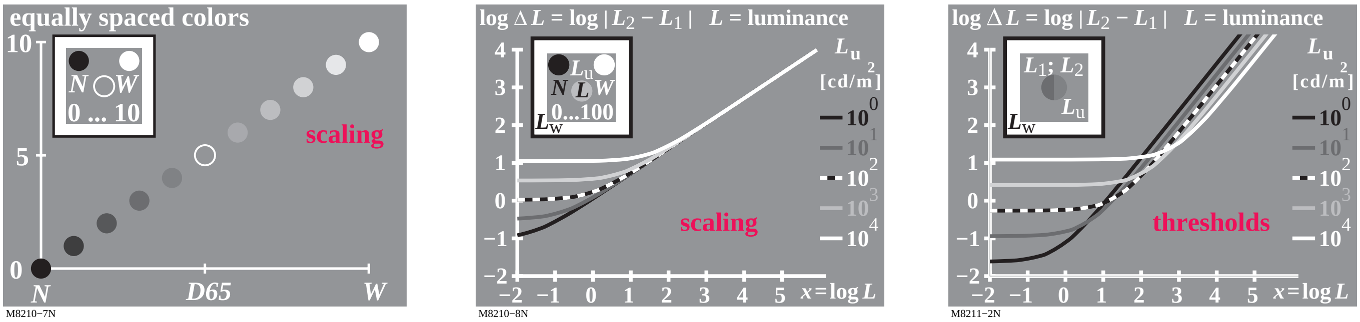

Figure 8: Experimental scaling results of a color series N – W and thresholds along this achromatic series

Experimental results on scaling along the luminance axis have been in agreement with the Munsell lightness scaling which is described by a cube root transformation of luminance. The plot in the middle shows additionally the dependence on the luminance level. The slop in a log–log plot is around 0.5. Our and other threshold experiments show a completely different slop of 1 in the right part of Fig. 8.

4.0 Modelling the receptor sensitivities by parables and the saturation by straight lines

The next figures show plots of physiological data. We model the receptor sensitivities including the V(l) function. The ratio of the sensitivity is used to calculate the saturation.

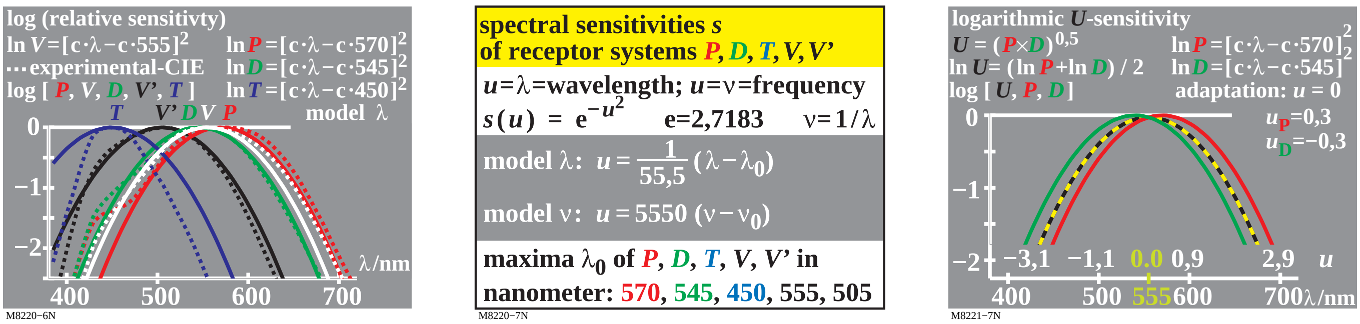

Figure 9: Modelling of receptor sensitivities by an exponential wavelength function which produce a parable for the sensitivity.

The receptor sensitivity of Pb(l) seems to have a hump in the blue region. This hump can be simulated by adding 1% of the blue sensitivity to the V(l) function (please read Pb(l) as P-bar-(lambda) and V(l) as V(lambda))

To get a good agreement to the threshold and scaling data we have simulated the V_{Judd}(l) function which is in the range between 400 nm and 480 nm by a factor 10 larger compared to photometric V(l) function. This simulation is a simulation of the P cone sensitivity and not a contribution of the blue cone to V(l). We use

$V_{Judd}(l) = V(l) + 0.01\ Tb(l)$ (4;1)

and for the tristimulus values

$Y_{Judd} = Y + 0.01\ Z$ (4; 2)

Instead of the ratio a = x / y = X / Y we will use

$a_{Judd} = X\ /\ Y_{Judd}$ (4; 3)

and similar for the ration b = – 0.4 z / y = – 0.4 Z / Y we will use

$b_{Judd} = -\ 0.4\ z\ /\ y_{Judd}$ (4; 4)

This reduces the distance from the achromatic point from (a=300, b=–1000) to (a_{Judd}=30, b=–100). This distance is called by *Evans* the “chromatic strength” of a wavelength. This distance can describe the experimental data of *Evans*

Basic data, methods and formula to bridge the gap for color differences

on absolute threshold data and colour of equal greyness (G_0-colours) of different wavelength much better if the V_{Judd}(l) is used.

Additionally we need such a change to describe scaling and threshold data in the blue-red area. In this area then the curvature of the blue hue, e. g. of the *Munsell* hue 5PB is reduced but still it shifts a lot towards red.

The red–green coordinate is defined as

$$a_{Judd} = x / y_{Judd} = X / Y_{Judd} \qquad (4;5)$$

1. If we neglect for a moment the hue difference between the P and the CIELAB a* axis then the red part of the xb(l) in this equation may be rewritten by the ratio of the sensitivity Pb(l) and Y_{Judd} (Simulation r=red)

$$a_{Judd,r} = a_P\, P / \; Y_{Judd} \qquad (4;6)$$

This simulation in the red part shifts the maximum of the xb(l) sensitivity from about 600 nm to a maximum of about 570 nm of the Pb(l) sensitivity. This reduces the ratio $a_{Jud,s1}$ at the red end of the spectrum by a factor 3.

2. If we neglect for a moment the hue difference between the D and the negative CIELAB a* axis then the green part of the xb(l) in this equation may be rewritten by the ratio of the sensitivity Db(l) and Y_{Judd} (Simulation g=green)

$$a_{Judd,g} = - a_D\, D / \; Y_{Judd} \qquad (4;7)$$

This simulation in the green part reduces $a_{Judd,g}$ at 500 nm to a negative value and at the blue end of the spectrum to 0.5. For this part near 400 nm it is approximately Y_{Judd} = Pb(l) = Db(l) and therefore $a_{Judd,g}$ is approximately zero but still b_{Judd} is around –100

$$b_{Judd,bj} = - b_T\, T / Y_{Judd} \qquad (4;8)$$

Therefore we expect that threshold discrimination may be described by the two axis

$$a_{Judd,r\text{-}g,\ threshold} = (a_P\, P - a_D\, D) / \; Y_{Judd} \qquad (4;9)$$

$$b_{Judd,b\text{-}j,\ threshold} = - b_T\, T / Y_{Judd} \qquad (4;10)$$

We may compare this result with the line drawn in Fig. XX, which cut the (*x,y*) chromaticity diagram near 400 nm and 555 nm. This line defines a constant P / D ratio. The MacAdam ellipses show a preference axis of the discrimination results along this line. The other preference axis is along the line which is defined by the ratio T / Y in Fig 22.

For scaling at least the P/D=const axis seem to shift to the axis defined by the ratio a = x / y in the (*x,y*) chromaticity diagram. This shift may be described by a blue contribution to the *a* axis. This blue contribution simulates the blue part of xb(l). So we may use the following two equations

$$a_{Judd,r\text{-}g,\ scaling} = (a_P\, P - a_D\, D + a_T\, T) / Y_{Judd} \qquad (4;11)$$

$$b_{Judd,b\text{-}j,\ scaling} = - b_T\, T / Y_{Judd} \qquad (4;12)$$

We can not expect linear equations for scaling and will use logarithmic (nonlinear) equations of a_{Judd} and b_{Judd} for this case and call the coordinates (a', b').

But in the red-green direction a linear equation may still be good enough. A linear function can be used instead of the logarithmic one because the coordinate range is only between about -1 and 1. For the blue axis the coordinate range is between about 1 to -100. So here the logarithmic function is essential for the b' axis.

$$a'_{Judd,r\text{-}g,\ scaling} = a'_P \log (P / Y_{Judd}) - a'_D \log (D / Y_{Judd}) + a'_T \log (T / Y_{Judd}) \qquad (4;13)$$

$$b'_{Judd,b\text{-}j,\ scaling} = - b'_T \log (T / Y_{Judd}) \qquad (4;14)$$

If in a special case $a'_P = a'_D = a'_{D+P}$ then

$$a'_{Judd,r\text{-}g,\ scaling} = a'_{P+D} \log (P / D) + a'_T \log (T / Y_{Judd}) \qquad (4;15)$$

$$b'_{Judd,b\text{-}j,\ scaling} = - b'_T \log (T / Y_{Judd}) \qquad (4;16)$$

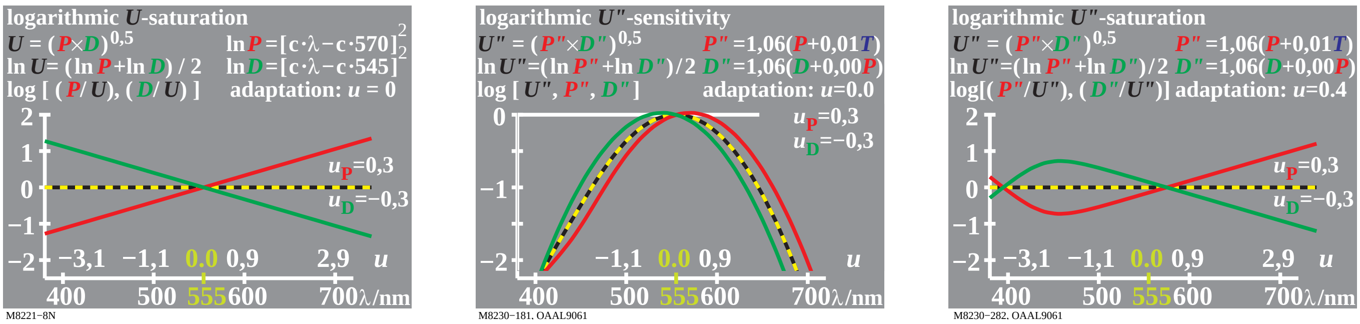

Figure 10: Saturation is a straight line (left) and 1% of T (middle) decreases saturation in the blue

If the P sensitivity includes 1% of blue sensitivity (shows the Judd hump of V(l) below 480 nm) then saturation decreases again in the blue

5.0 Modelling the threshold and scaling data as function of sample luminance

Now we construct a model for the physiological signals and look how they change as function of chromaticity and luminance of the stimuli. Later we will discuss the physiological signals for the complementary optimal colours.

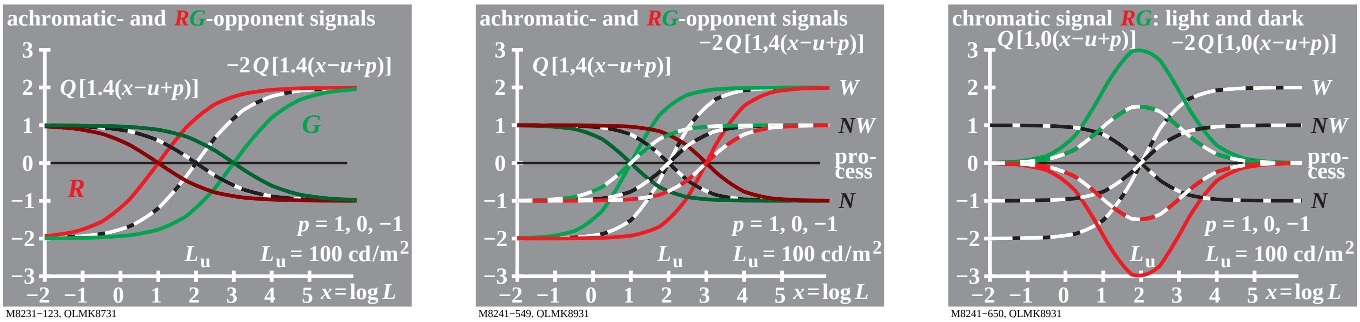

Figure 11: Processes White W and Black N and sum NW and high discrimination for threshold process W

Fig. 11 shows two opponent processes White W and Black N. Light and dark colours are used for the coding in this and the following figures. The slope and amplitude seems to change according to the temporal, spatial and luminance changes. We have used an **amplitude and slope** of **one log unit and –0.5 for the Black process N** and **two log units and 1.0 for the White process W**.

> Remark: Normal colour vision in offices shows samples with a luminance factor between 2.5 and 90 in a grey background with the luminance factor of 20. If the black line is the basic line for grey surround then this range is totally within the +/– one log unit range of the black process N.

It is known that there are different opponent processes for spacial and temporal interaction of the receptive fields. Opponent processes as function of luminance have been studied by Lee. Our signals look similar to many of his results.

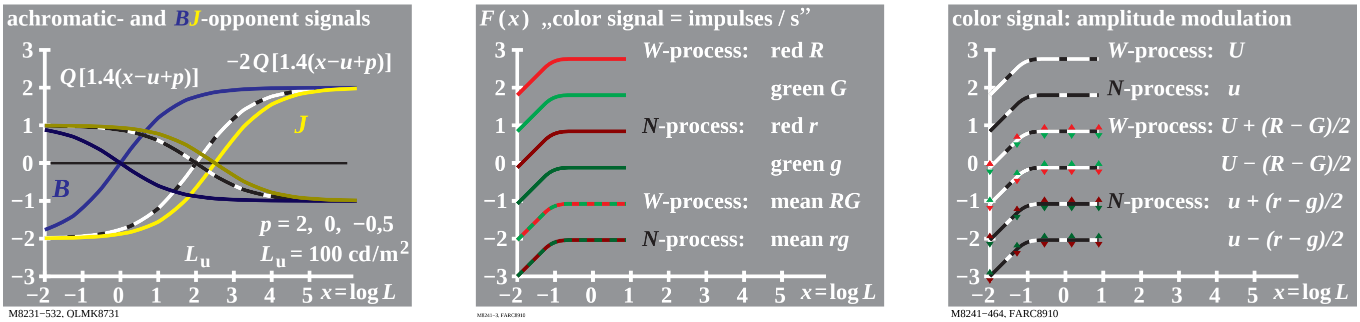

Figure 12: Blue – Yellow opponent signals and process colour coding for the figures

The black–Yellow signals may be in a first step asymmetric because of the large saturation for blue and the low saturation for yellow. The saturation p shifts here on the luminance scale by –2.0 log units for blue and 0.5 log units for Yellow. It is shown in the coding list that this asymmetric property may be changed at the receptor level by an amplitude modulation on top of the achromatic signal. This makes the signals symmetric. There is another way to calculate these signals by using the sensitivity Nb(l) which is the log mean of the Tb(l) and V(l) sensitivity (compare Fig. 15).

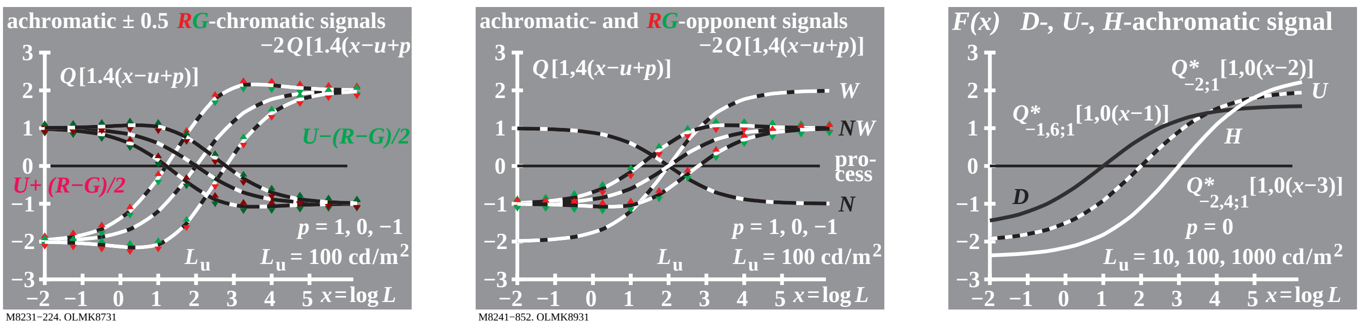

Figure 13: Amplitude modulation discrimination of achromatic colours for three adaptation luminances

Basic data, methods and formula to bridge the gap for color differences

By amplitude modulation the signals as function of luminance are always symmetric compared to the achromatic signals for the three processes Black N, White W and the sum NW. The figure on the right side shows the possible change of the achromatic signal with the adapting luminance. The amplitude is increasing by a slope of 1/6 with luminance.

6.0 Modelling the threshold and scaling data as function of sample and adaptation luminance

Fig. 13 already includes the change of signal (and discrimination and scaling) of samples for three adaptation luminances between 10 cd/m^2 and 1000 cd/m^2

We will have a brief study of the colour case. Experiments show that the maximum of discrimination will not change to darker or lighter colours for the two opponent colours. The summation of the signals for the black and white process may be the first step. Then a summation of the colour opponent processes produce signals which are very similar to the achromatic signals (see Fig. 14 middle)

It remains an open question in which sequence the different processes contribute to the colour vision process.It is obvious that the sum and the differences of the processes play the important role. This has been recently reported by the chairman E. Martinez-Uriegas of CIE TC1-60 about chromatic and achromatic contrast sensitivity functions (CSF).

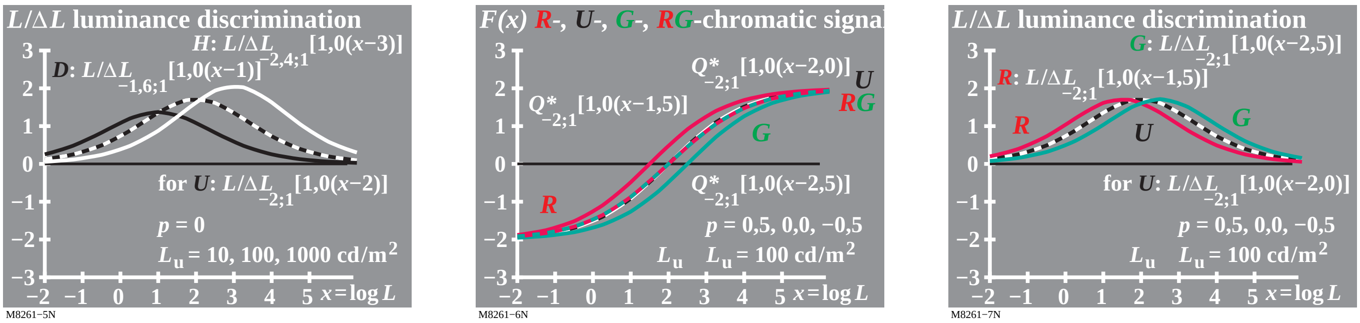

Figure 14: Luminance discrimination of achromatic and chromatic colours).

Therefore the expected **luminance discrimination** in the right figure is not true. The luminance discrimination is nearly the same as for achromatic colours. Therefore there must be a very fast summation of the R and G response (see middle) which gives the required sum (similar to the achromatic signal) and then a similar discrimination compared to the achromatic case. There may be small differences compared to the achromatic case which increase with the saturation of the opponent colours.

6.0 Model for symmetric saturation

If the two opponent processes have very different saturation, e. g. in the blue yellow case, then the difference signals, e. g. either 0.5 (R-G) or 0.5 (Y-B), seem to be added on the achromatic signal. If we follow this assumption then mathematically one can use the mean log sensitivity of Ub(l) and Tb(l) to calculate the saturation differences.

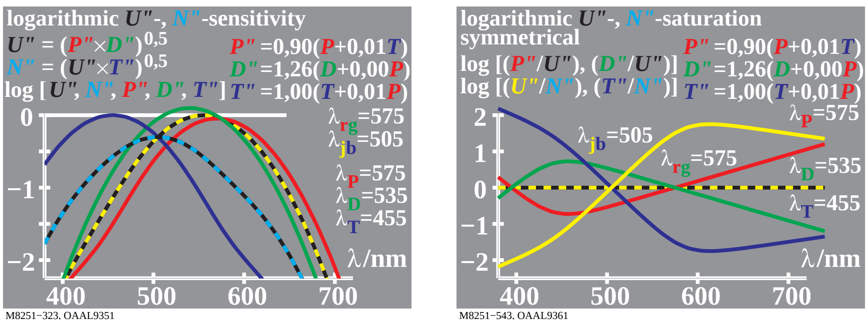

Figure 15: Definition of logarithmic sensitivities and symmetric saturation used for the calculation of diagrams (a’, b’)

The maximum sensitivity of Nb(l) is at 505 nm which is in the middle between 555 nm and 455 nm. Fig. 15 shows this sensitivity with a blue-green–black colour coding. The yellow–blue ratios U”/N” and T”/N” and the red-green ratios are symmetric. One can use one of both to construct a diagram similar to (*a’*,*b’*)

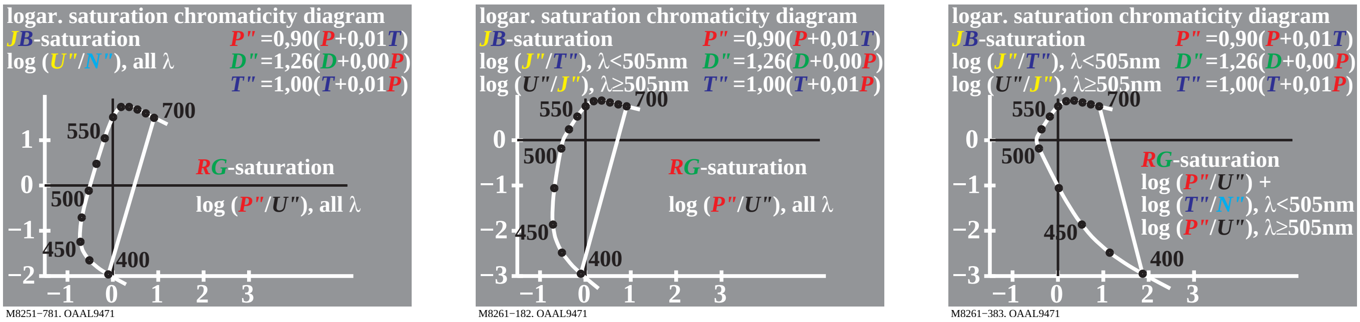

Figure 16: Logarithmic diagrams (a', b') with axis for threshold and scaling

Fig. 16 uses on the left side symmetric saturation components in red-green and yellow-blue direction. This symmetric saturation is necessary to describe the Holtsmark-Valberg threshold results for complementary optimal colours. In the middle the yellow-blue component is asymmetric and the red-green symmetric. This asymmetry may arise by an additional summation of Nb(l) and Ub(l) with a maximum sensitivity at 530 nm [=0.5*(505+555)]. On the right side a blue contribution T/U has shifted the unique hues around 575nm and 475 nm on the vertical axis.

The Holtsmark-Valberg results and the preliminary interpretation do not decide if a linear or logarithmic saturation must be used. If the chromatic value A_{01} is constant for complementary colours then also any (logarithmic) function of A_{01} produce constant data (compare eqation 2;10)

7.0 Transformation from CIEXYZ to CIELAB and vice versa

color space CIELAB 1976, color values, -attributes, -coordinates *(a', b')*

tristimulus values *X, Y, Z* −> color coordinates *L*, a*, b**

lightness	$L^* = 116\,(Y/Y_n)^{1/3} - 16$	
RG−chromaticness	$a^* = 500\,[\,(X/X_n)^{1/3} - (Y/Y_n)^{1/3}\,] = 500\,[\,a' - a'_n]\,Y^{1/3}$	
JB−chromaticness	$b^* = 200\,[\,(Y/Y_n)^{1/3} - (Z/Z_n)^{1/3}\,] = 500\,[\,b' - b'_n]\,Y^{1/3}$	

color coordinates *L*, a*, b −> tristimulus values *X, Y, Z***

tristimulus values

$$X = X_n\,[\,(L^* + 16)/116 + a^*/500\,]^3$$
$$Y = Y_n\,[\,(L^* + 16)/116\,]^3$$
$$Z = Z_n\,[\,(L^* + 16)/116 - b^*/200\,]^3$$

coordinates (a',b') for CIELAB 1976, LABHNU 1977, LABHNUx 1979

CIELAB 1976, 2°	$a' = 0{,}2191\,(x/y)^{1/3}$	$b' = -\,0{,}08376\,(z/y)^{1/3}$
LABHNU 1977	$a' = (x/y + 1/6)^{1/3} / 4$	$b' = -\,(z/y + 1/6)^{1/3} / 12$
LABHNU1 1979	$a' = (x/y + 1) / 15$ *linear!*	$b' = -\,(z/y + 1/6)^{1/3} / 12$
LABHNU2 1979	$a' = (x/y + 1/6)^{2/3} / 15$	$b' = -\,(z/y + 1/6)^{1/3} / 12$
CIELAB 1976, 10°	$a' = 0{,}2193\,(x_{10} / y_{10})^{1/3}$	$b' = -\,0{,}08417\,(z_{10} / y_{10})^{1/3}$
constants for CIELAB, 2°,10°	$a_2 = 500\,(1/X_n)^{1/3} = 0{,}2191$ $a_{10} = 500\,(1/X_{n10})^{1/3} = 0{,}2193$	$b_2 = -\,200\,(1/Z_n)^{1/3} = -\,0{,}08376$ $b_{10} = -\,200\,(1/Z_{n10})^{1/3} = -\,0{,}08417$

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Figure 17: Transformation from X,Y,Z to L*a*b* and vice versa.

The (*a',b'*) diagram is very useful to calculate the coordinates *a** and *b** of the CIELAB colour space. A cube root transformation of the chromaticity diagram ($a = x / y$, $b = -0.4\, z / y$) is used here. In the red–green direction a linear coordinate

$$a' = (x / y + 1) / 15$$

may be used to describe the Munsell and OSA scaling.

7.1 Scaling data of the OSA and Munsell system in different diagrams

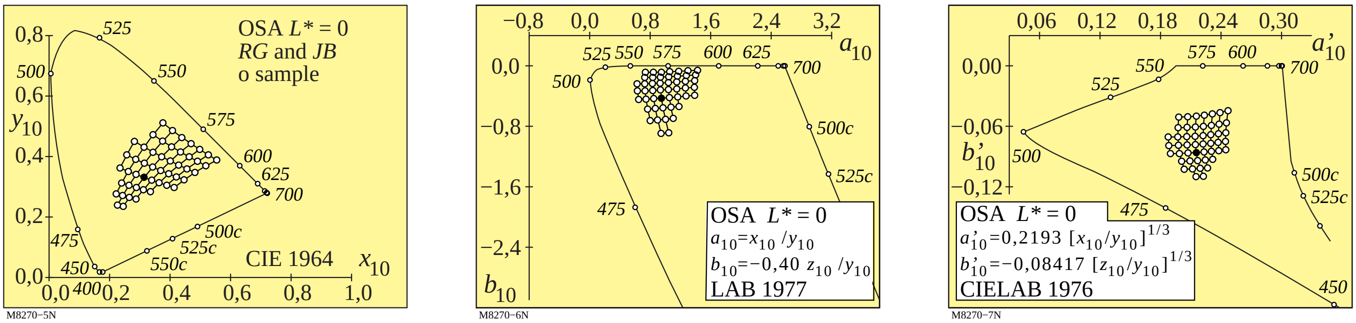

Figure 18: OSA colour samples in the chromaticity diagrams (*x*,*y*), (*a*,*b*) and the diagram (*a'*,*b'*)
The data of the OSA lightness L_{OSA}=0 are plotted. The colours with open circles are realized. In the diagram (*a'*,*b'*) the colours are approximately on a regular grid.

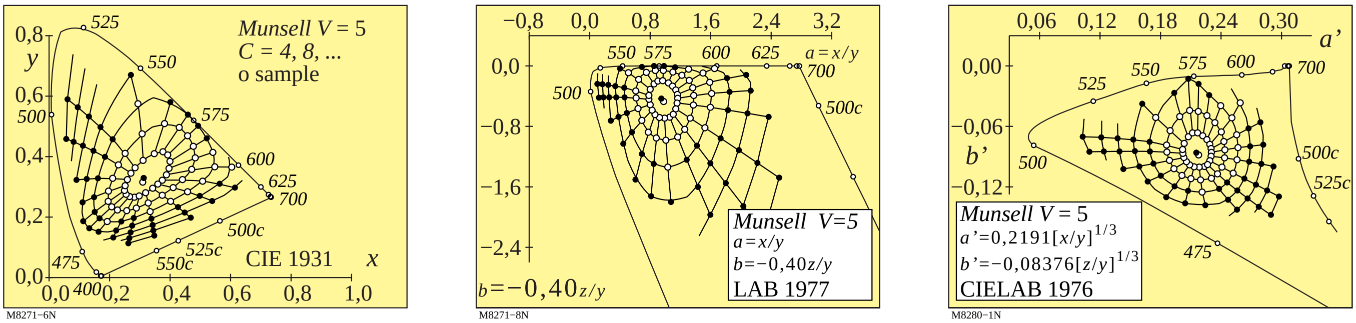

Figure 19: Munsell colour samples in the chromaticity diagrams (*x*,*y*), (*a*,*b*) and the diagram (*a'*,*b'*)
The data of Munsell value V=5 are plotted. The colours with open circles are realized. In the diagram (*a'*,*b'*) the colours are approximately on circles.

7.2 MacAdam and BAM thresholds in the chromaticity diagrams (*x*,*y*), (*a*,*b*) and the diagram (*a'*,*b'*)

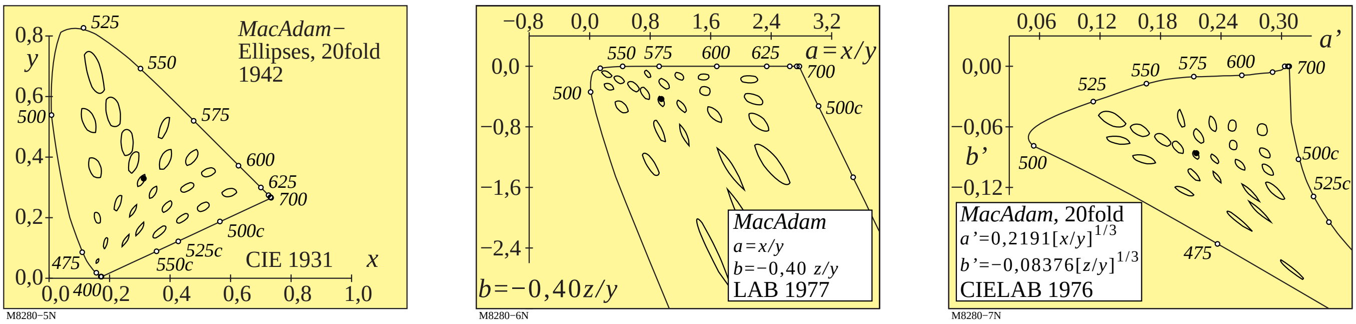

Figure 20: MacAdam ellipses in the chromaticity diagrams (*x*,*y*), (*a*,*b*) and the diagram (*a'*,*b'*)
The MacAdam ellipses are very different in size in the diagram (*a'*,*b'*). We will need an other diagram (see Fig. 22) to produce approximately circles of equal size.

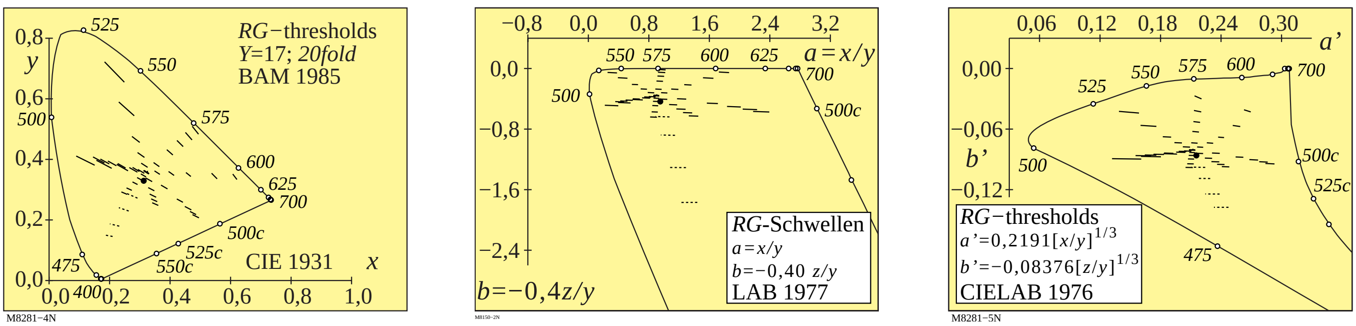

Figure 21: BAM thresholds in the chromaticity diagrams (*x*,*y*), (*a*,*b*) and the diagram (*a'*,*b'*)

The BAM experimental results are in red-green direction very similar to the MacAdam results. The are other BAM results for the vertical and the two diagonal directions (see Richter).

7.3 Analysis of threshold data

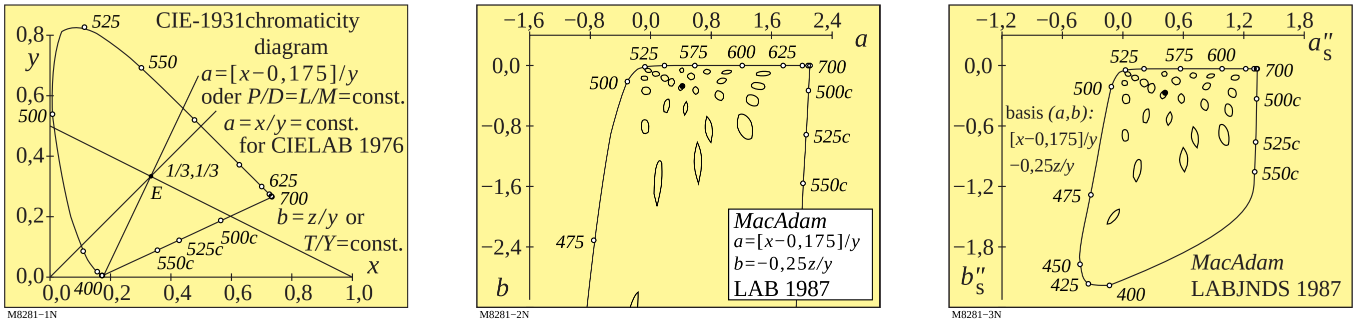

Figure 22: Analysis of threshold axis in chromaticity diagram and transform

There are two main axis for both threshold and scaling differences. One threshold axis is (see Fig. 22, left)

$$a = [\,x - 0.175\,] / y$$

$$= [\,x - 0.175\,(x + y + z)\,] / y = (\,0.825\,y - 0.175\,z - 0.175\,y) / y$$

The constant 0.175 may be deleted if chromaticity differences are calculated

$$a = (\,0.825\,y - 0.175\,z\,) / y$$

The chromaticity diagram (a,b) has now a spectrum locus for purple colours which is a vertical line. Both axis are shrinked in Fig. 23 (right) to get colours of equal discrimination to about the same size.

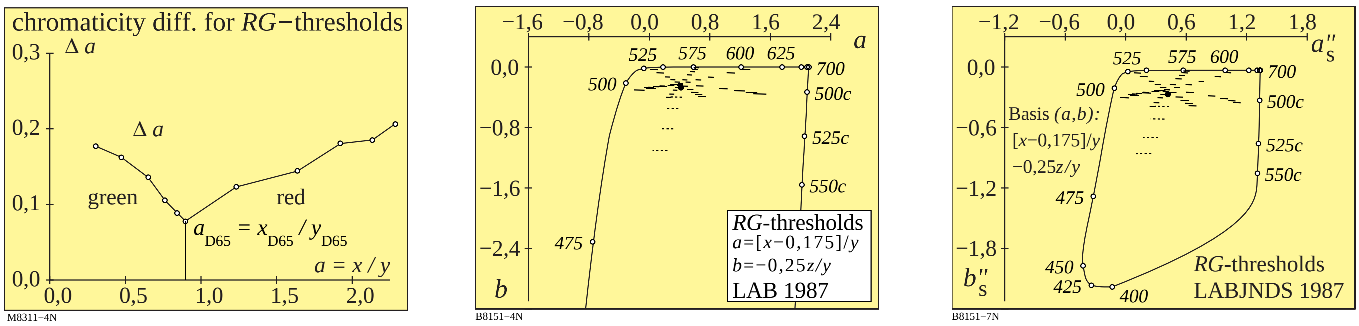

Figure 23: Analysis of threshold along the red–green axis in chromaticity diagram (*a*,*b*) and transform

The following equation is used for the change of the coordinate a_{01} in the right part of Fig. 22 and 23

$$a''_{01} = (a_{01} - 1) / [\,1 + 0.5\ \ |\,a_{01}\ - 1|\,]$$

7.4 Equations for the description of threshold data

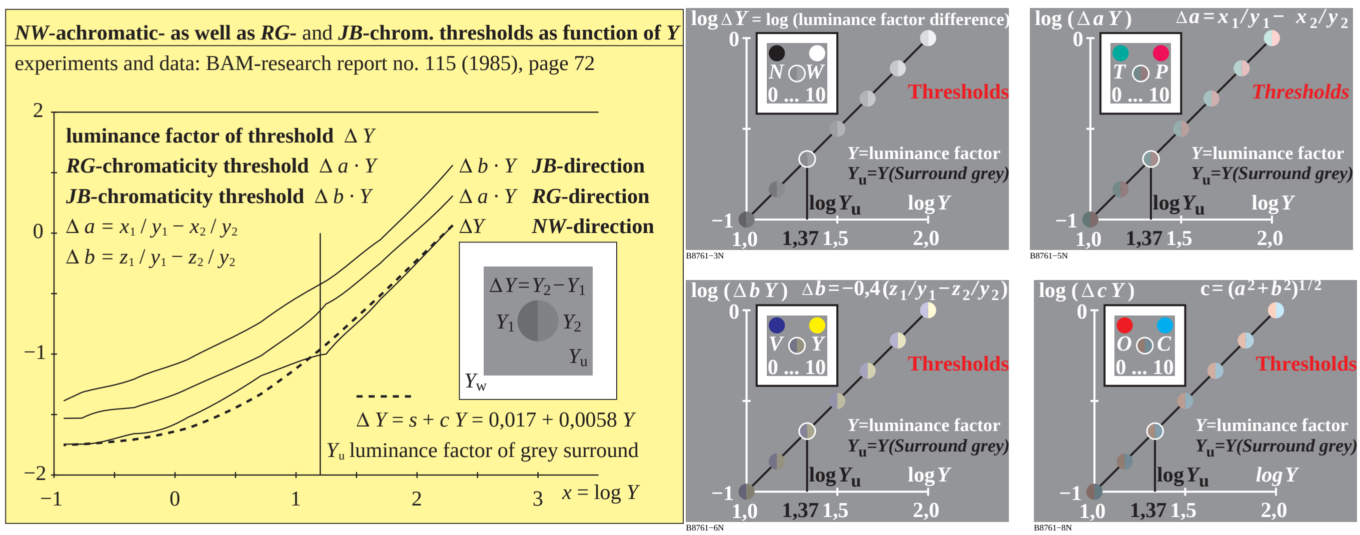

Figure 24: Analysis of threshold change as function of luminance and chromaticity (*a,b*)

The figure describes the luminance factor threshold delta Y and the chromaticity threshold delta a and delta b as function of luminance factor. For compensatory colours of equal luminance we see

$$\log [(\text{delta } a)\, Y\,] = \log Y$$

This leads to (compare Fig. 23)

$$\text{delta } a = \text{const.}$$

The threshold circles in the chromaticity diagram (*a, b*) are according to this figure constant for about the luminance factor range between 10 and 100 for a mean grey surround with Y=20. The diameter increases for dark colours.

There is a continuous decrease by the scaling data of the Munsell between Value 2 and Value 9 and similar for the OSA system. So we have here a main difference between scaling and threshold data as function of the sample luminance factor.

8.0 Summary

Some basic results of a paper “Cube root color spaces and chromatic adaptation” published in 1980 in Color, Research and Application and new experimental data and models are useful to bridge the gap between threshold and large CIELAB colour differences.

It is the aim of the new model not to change the main properties of the CIELAB colour space which is recommended and used for large colour differences in the colour reproduction area nor the main properties of the CIE colour difference formulas which are recommended for up to 5 CIELAB units and based on super threshold (pass-fail) experiments. The improved model is based on experimental threshold results of e. g. MacAdam, Richter, and Holtsmark-Valberg and others.

The metric for the description of the colour threshold and the colour scaling data is very different. The first is in some parts a linear metric and the second at least in the blue yellow direction a very nonlinear metric. Main properties of the metric are symmetric for complementary optimal colours which lead to a special structure of the colour difference formulas.

Two opponent colour processes a black process N and a white process W produce signals with different slopes (-0.5 and 1) and different amplitudes (1 and 2) in log– log plot as function of sample luminance. This makes the threshold process White W the most sensitive compared to the summation process NW which describes scaling. So if the process W is at threshold then the process NW for scaling is below threshold and does not contribute to the colour difference. In the range of about two thresholds the scaling process starts to contribute to the colour difference.

For the larger luminance factor range a more complex S-shaped function of a physiological model (see Fig. 1) is necessary. The slopes of these functions are 0.5 and 1 similar as in a recent paper of Vienot (2003). The luminance range is by a factor two different for these functions. This seem similar in the spatial and temporal range discussed recently by Martinez-Uriegas (2003).

9.0 References

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For further information see e. g. the URL
http://www.ps.bam.de

Annex A: Formulas of Richter (1979, 1985, 1996) to describe scaling and threshold data

color space CIELAB 1976, color values, -attributes, -coordinates *(a’, b’)*

tristimulus values *X, Y, Z* –> color coordinates *L*, a*, b**

lightness	$L^* = 116\,(Y/Y_n)^{1/3} - 16$
RG−chromaticness	$a^* = 500\,[\,(X/X_n)^{1/3} - (Y/Y_n)^{1/3}\,] = 500\,[\,a' - a'_n]\,Y^{1/3}$
JB−chromaticness	$b^* = 200\,[\,(Y/Y_n)^{1/3} - (Z/Z_n)^{1/3}\,] = 500\,[\,b' - b'_n]\,Y^{1/3}$

color coordinates *L*, a*, b –> tristimulus values *X, Y, Z***

tristimulus values

$$X = X_n\,[\,(L^* + 16)/116 + a^*/500\,]^3$$
$$Y = Y_n\,[\,(L^* + 16)/116\,]^3$$
$$Z = Z_n\,[\,(L^* + 16)/116 - b^*/200\,]^3$$

coordinates (a’,b’) for CIELAB 1976, LABHNU 1977, LABHNUx 1979

CIELAB 1976, 2°	$a' = 0{,}2191\,(x/y)^{1/3}$	$b' = -\,0{,}08376\,(z/y)^{1/3}$
LABHNU 1977	$a' = (x/y + 1/6)^{1/3} / 4$	$b' = -\,(z/y + 1/6)^{1/3} / 12$
LABHNU1 1979	$a' = (x/y + 1) / 15$ *linear!*	$b' = -\,(z/y + 1/6)^{1/3} / 12$
LABHNU2 1979	$a' = (x/y + 1/6)^{2/3} / 15$	$b' = -\,(z/y + 1/6)^{1/3} / 12$
CIELAB 1976, 10°	$a' = 0{,}2193\,(x_{10} / y_{10})^{1/3}$	$b' = -\,0{,}08417\,(z_{10} / y_{10})^{1/3}$
constants for CIELAB, 2°,10°	$a_2 = 500\,(1/X_n)^{1/3} = 0{,}2191$ $a_{10} = 500\,(1/X_{n10})^{1/3} = 0{,}2193$	$b_2 = -\,200\,(1/Z_n)^{1/3} = -\,0{,}08376$ $b_{10} = -\,200\,(1/Z_{n10})^{1/3} = -\,0{,}08417$

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Fig. A.1: Transformation between *X,Y,Z* and *L*a*b and vice versa and different coordinates (*a’,b’*)**
An (*a’,b’*) diagram is useful to calculate the chroma coordinates *a** and *b** of the CIELAB colour space. A cube root transformation of the chromaticity diagram ($a = x / y$, $b = -\,0.4\,z / y$) is used here. In the red-green direction a **linear coordinate** *a* which is **proportional to the ratio *x / y*** produces a high correlation for the experimental results

color threshold formula LABJNDS 1985 (JND=just noticeable difference)

$$\Delta E^*_{JND} = Y_0\,[\,(\Delta Y)^2 + (\,a_0\,\Delta a'' \cdot Y)^2 + (\,b_0\,\Delta b'' \cdot Y)^2]^{1/2} \,/\, (\,s + d\,Y^e)$$

$a = x / y \quad a_n = x_n / y_n \quad b = -\,0{,}4\,z / y \quad b_n = -\,0{,}4\,z_n / y_n$

$a'' = a_n + (\,a - a_n) / (\,1 + 0{,}5\,|a - a_n|\,)$

$b'' = b_n + (\,b - b_n) / (\,1 + 0{,}5\,|b - b_n|\,)$

n = *D65 or A (surround)*

$Y = (\,Y_1 + Y_2\,) / 2 \quad \Delta Y = Y_1 - Y_2 \quad \Delta a'' = a''_1 - a''_2 \quad \Delta b'' = b''_1 - b''_2$

$s = 0{,}0170 \quad d = 0{,}0058 \quad e = 1{,}0$

$a_0 = 1{,}0 \quad b_0 = 1{,}8 \quad Y_0 = 1{,}5$ *surround D65*

$a_0 = 1{,}0 \quad b_0 = 1{,}7 \quad Y_0 = 1{,}0$ *surround A*

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Fig. A.2: Transformation between *X,Y,Z* and color thresholds coordinates (*a”, b”*)
An (*a”, b”*) diagram is useful to calculate the threshold differences (JND = just noticeable differences). Some constants are given which describe the experimental results of Fig. 3.

Annex B: Some mathematics which may be useful to create the line element for the threshold data

Line element as function of the luminance factor *Y*

We calculate the deviation of the following threshold function Q^*_Y which depends only on luminance factor Y

$$Q^*_Y = \text{const}\ ln\ (1 + c_Y\ Y)$$

$$dQ^*_Y\ /\ dY = \text{const}\ /\ (1 + c_Y\ Y)$$

for dQ^*_Y = const:

$$dY = \text{const}\ (1 + c_Y\ Y)$$

Remark: For $c_Y\ Y >> 1$ we get the Weber-Fechner law $dY\ /\ Y$ = const

Line element as function of chromatic value *A*

We calculate the deviation of the following threshold function Q^*_A which depends only on chromatic value A

$$Q^*_A = \text{const}\ ln\ (1 + c_A\ A)$$

$$dQ^*_A\ /\ dA = \text{const}\ /\ (1 + c_A\ A)$$

for dQ^*_A = const:

$$dA = \text{const}\ (1 + c_A\ A)$$

Remark: For $c_A\ A >> 1$ we get (a possible new law) $dA\ /\ A$ = const

Line element as function of both chromaticity *a* and luminance factor *Y*

We calculate the deviation of the following threshold function Q^*_{aY} which depends on both chromaticity a and luminance factor Y

$$Q^*_{aY} = \text{const}\ ln\ (1 + c_{aY}\ a\ Y)$$

$$dQ^*_{aY}\ /\ da = \text{const}\ Y\ /\ (1 + c_{aY}\ a\ Y)$$

$$dQ^*_{aY}\ /\ dY = \text{const}\ a\ /\ (1 + c_{aY}\ a\ Y)$$

for dQ^*_{aY} = const and deviation to the chromaticity a

$$da = \text{const}\ (1 + c_A\ a\ Y)\ /\ Y$$

Remark: For $c_A\ a\ Y >> 1$ we get (a possible new law) $da\ \ Y\ /\ (a\ \ Y) = da\ /\ a$ = const

for dQ^*_{aY} = const and deviation to the luminance factor Y

$$dY = \text{const}\ (1 + c_A\ a\ Y)\ /\ a$$

Remark: For $c_A\ a\ Y >> 1$ we get the Weber-Fechner law $dY\ \ /\ Y$ = const